SWITCHING TECHNIQUES

- **A generic router model**
- **Three layers in an interconnection network**
  - **Routing layer:**
    make routing decision at intermediate router and establish the path through the network.
  - **Switching layer:**
    use physical layer protocols to implement mechanisms for forwarding messages through the network.
  - **Physical layer:**
    transfer messages and manage the physical channels between adjacent routers.

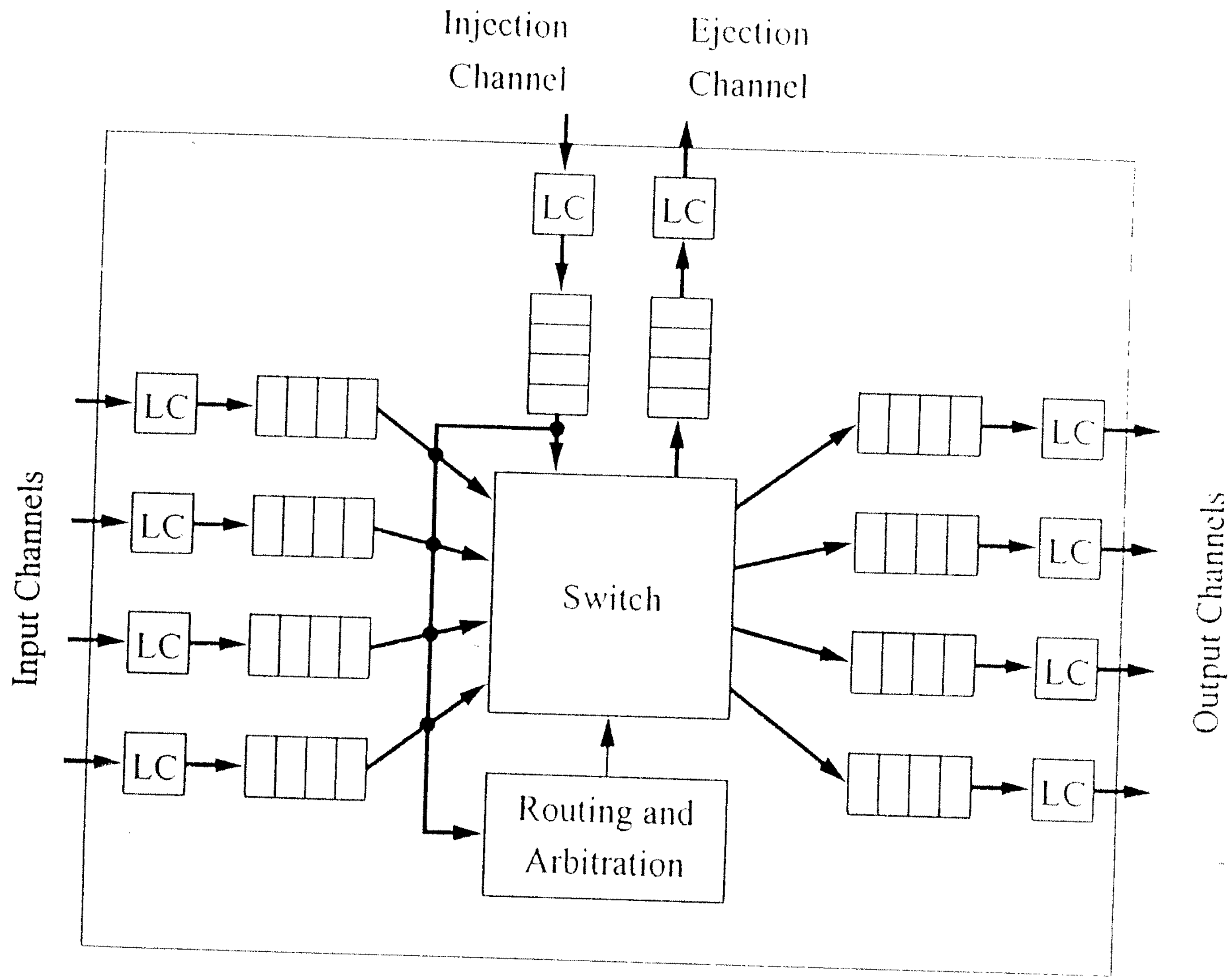


Figure 2.1. Generic router model. (LC = Link controller.)

- **Switching techniques determine**
  1. **when and how internal switches are set to connect router inputs to outputs;**
  2. **the time at which messages may be transfered along these paths.**
- **Assumptions:**
  - **Consider L-bit message in the absence of any traffic**
  - **Channel width: W bits**
  - **Message size: $L+W$ bits (message+ header)**
  - **Routing decision time: t_r sec.**
  - **Physical channel bandwidth: BW bits/sec.**
  - **Propagation delay of one channel: $t_w = \frac{1}{B}$**
  - **Switching delay (the delay inside the router): t_s**
  - **Source and destination are D links apart**

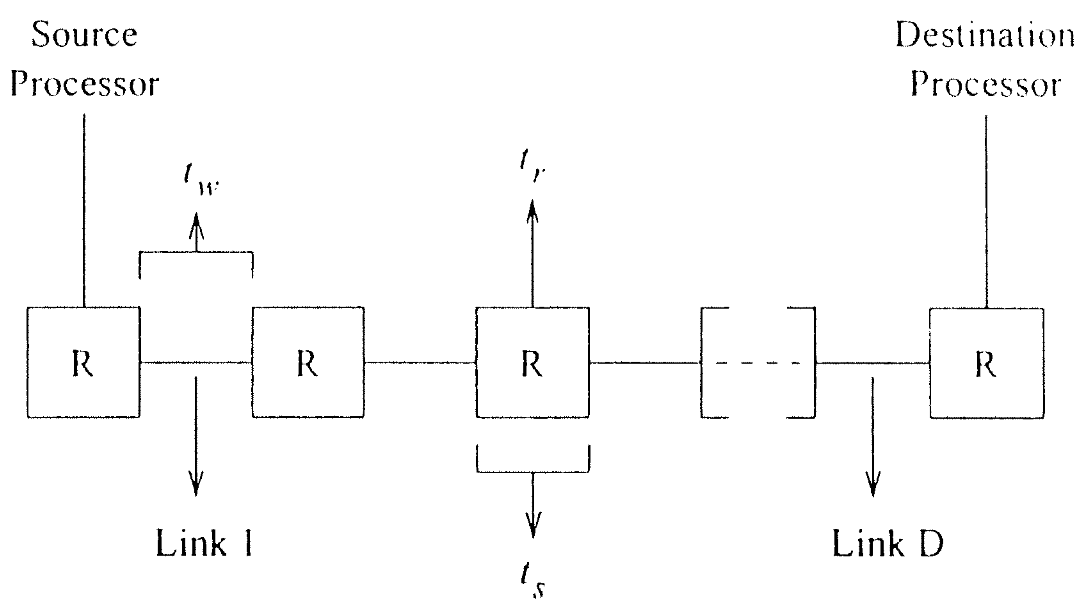


Figure 2.5. View of the network path for computing the no-load latency. (R = Router.)

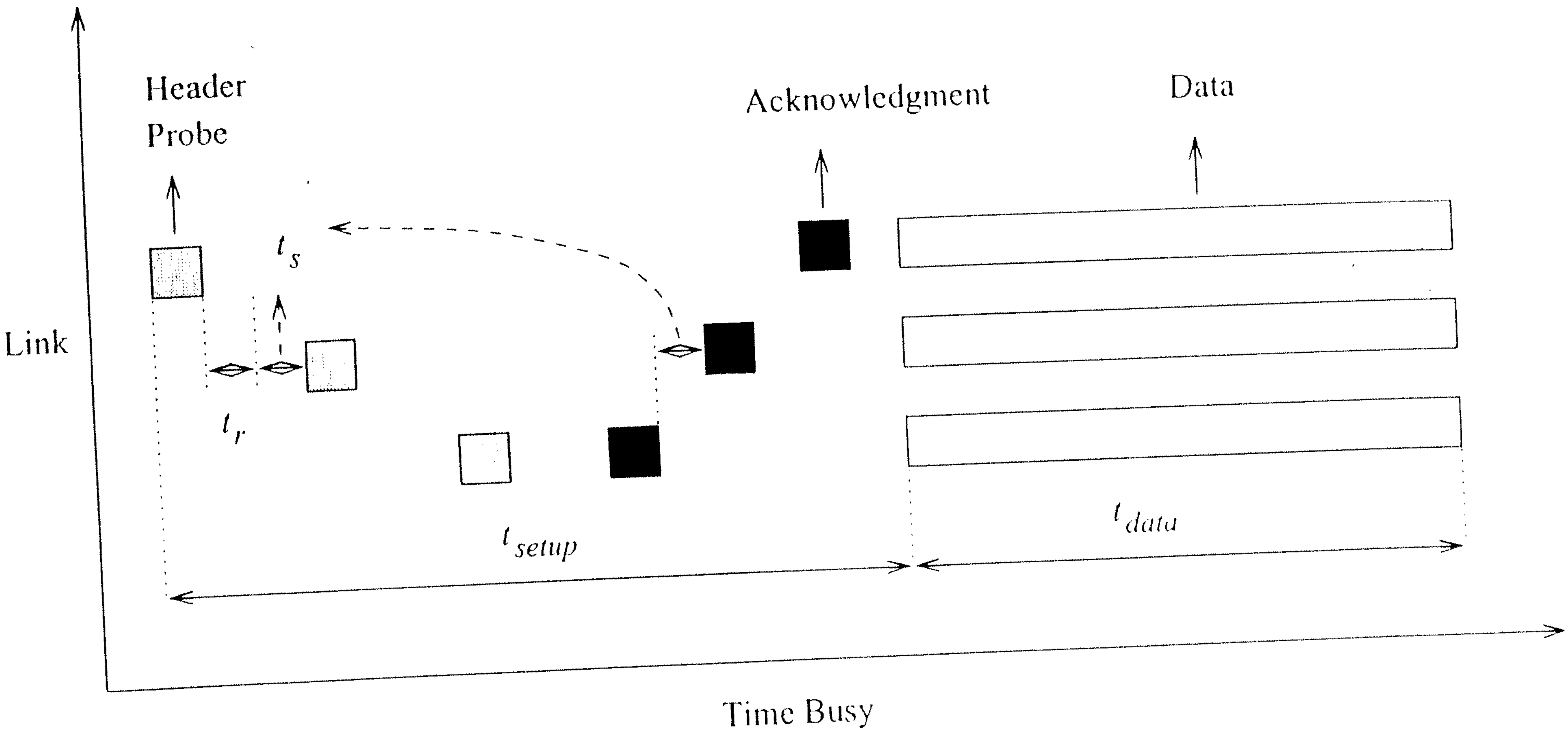


Figure 2.6. Time-space diagram of a circuit-switched message.

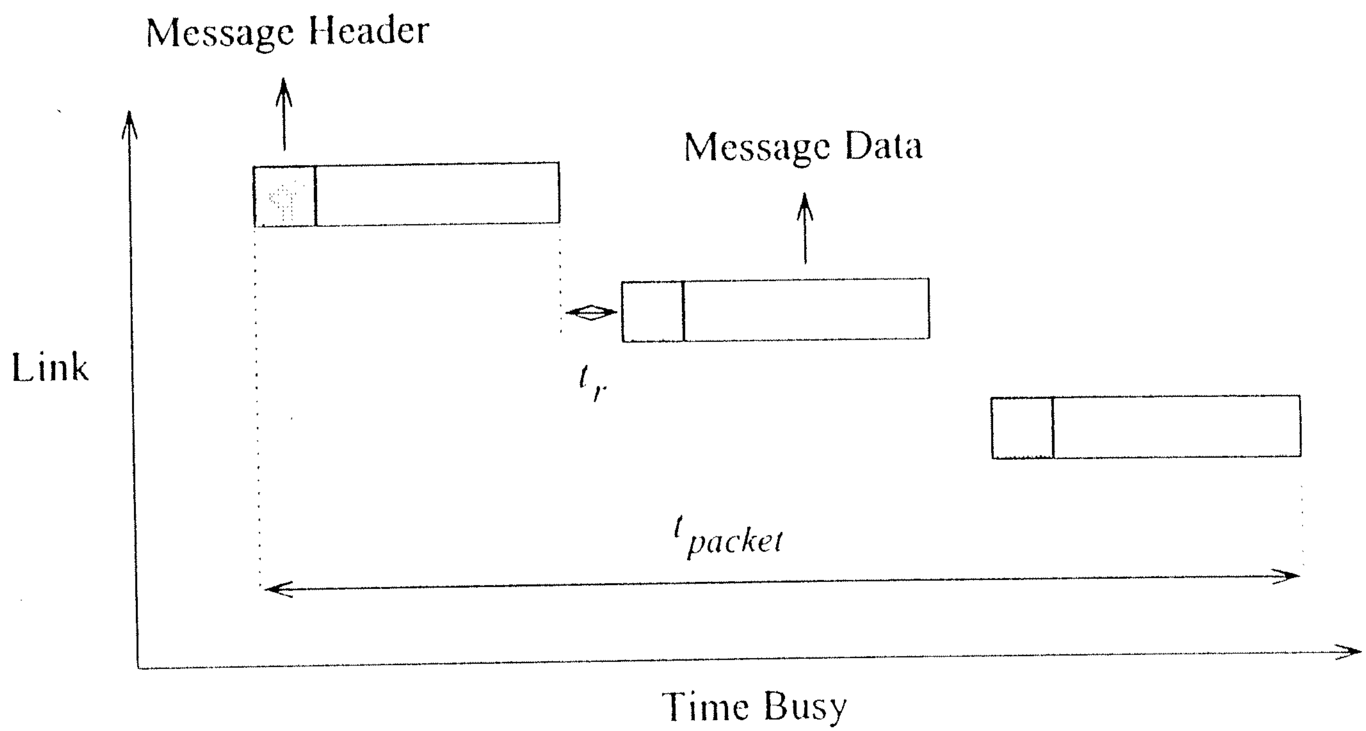


Figure 2.8. Time-space diagram of a packet-switched message.

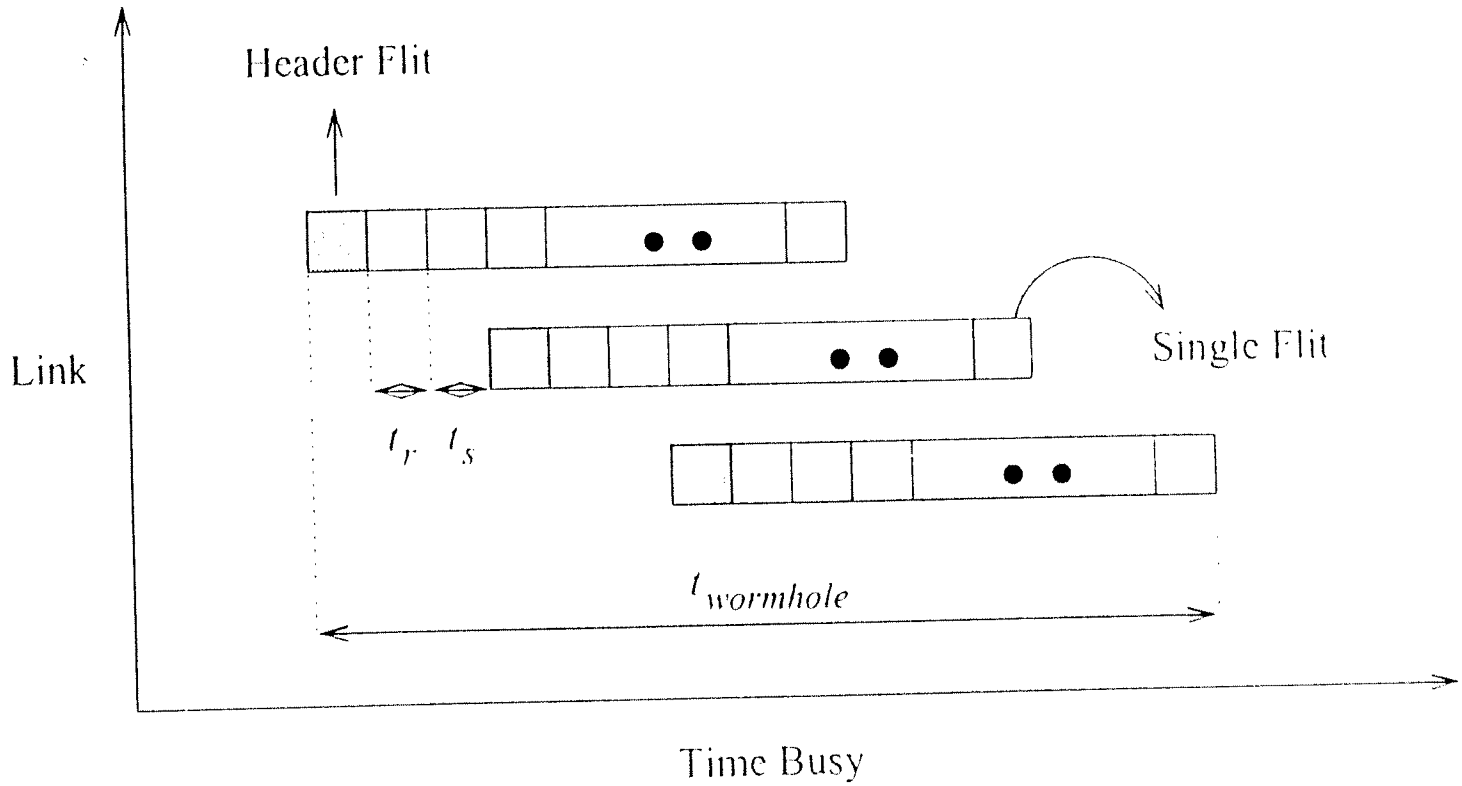


Figure 2.11. Time-space diagram of a wormhole-switched message.

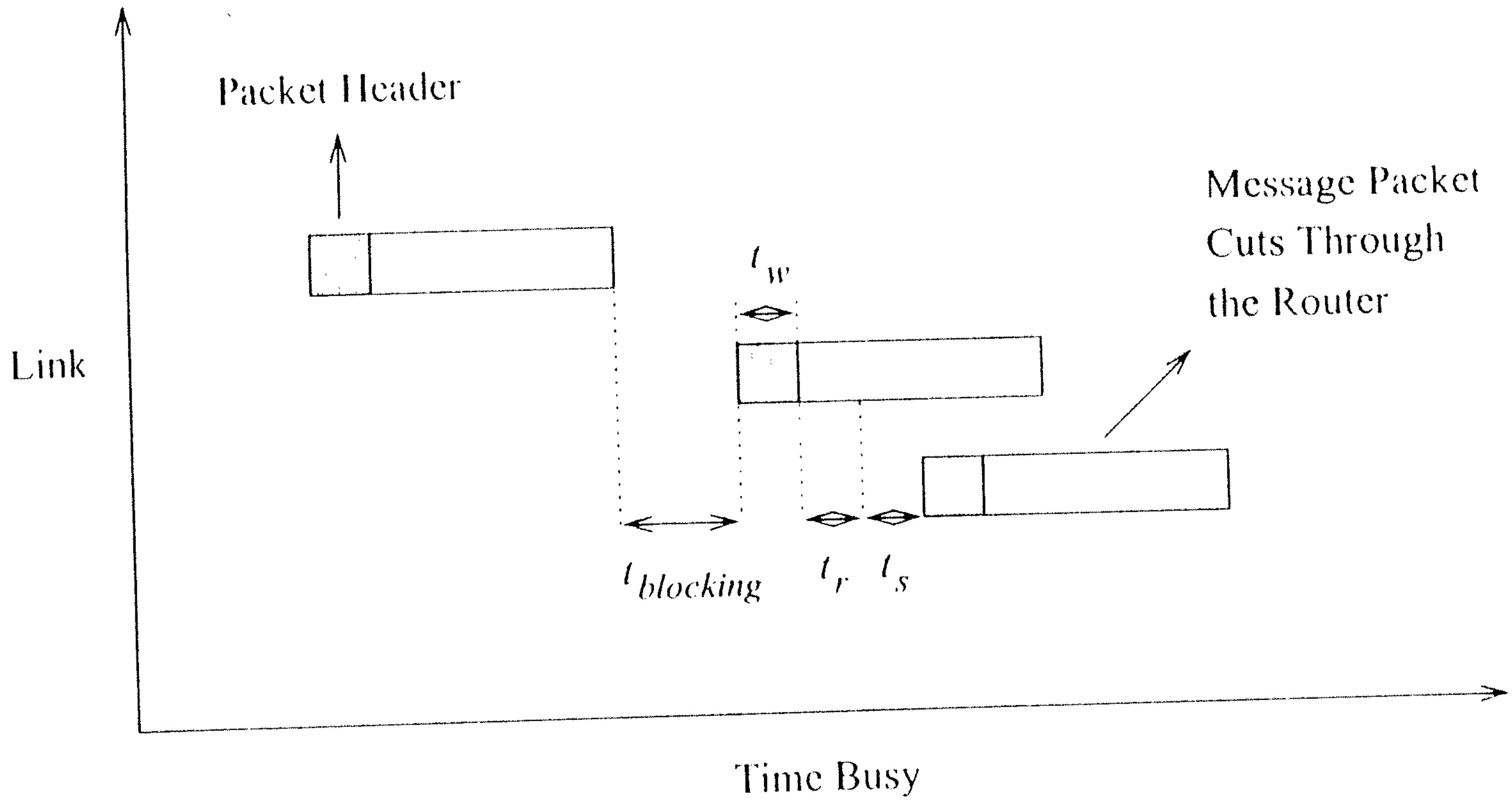


Figure 2.10. Time-space diagram of a virtual cut-through switched message. ($t_{blocking}$ = Waiting time for a free output link.)

- **Basic switching techniques**
  - **Circuit switching:**

    A physical path from the source to the destination is established and the switches on the path remain in their specified states until the path is released.

    How it works:

    * **Establish the path by a routing probe**
    * **Destination sends an acknowledgement**
    * **Transmit data**
    * **Release the path by destination or last few bits of the message**

    Latency:

$$t_{circuit} = t_{setup} + t_{data}$$

$$t_{setup} = D[t_r + 2(t_s + t_w)]$$

$$t_{data} = \frac{1}{B}\left\lceil\frac{L}{W}\right\rceil$$

Suitable for infrequent, long messages.

– **Packet switching:**

A packet (a group of bits of fixed length) moves from node to node, releasing links and switches immediately after using them. Also called store and forward switching.

How it works:

- **Message is divided into fixed-length packets**
- **Each packet contains routing information (in its header) and is routed individually.**
- **A packet is completely buffered at each intermediate node.**
- **Latency is proportional to the distance between source and destination.**

Latency:

$$t_{packet} = D\left[t_r + (t_s + t_w)\left\lceil\frac{L+W}{W}\right\rceil\right]$$

Suitable for frequent, short messages.

- **Worm-hole switching:**

  Pipelined (hardware) packet switching. A compromise between packet switching and circuit switching.

  How it works:

  * **Divide a packet into flits.**
  * **Only header flit contains the routing information and all flits in a packet follows the same path.**
  * **Only buffer a few flits at each router (not the entire packet).**
  * **In the case of blocking, message blocked in place.**

  Latency:

$$t_{wormhole} = D(t_r + t_s + t_w) + \max(t_s, t_w) \left\lceil \frac{L}{W} \right\rceil$$

– **Virtual cut-through:**

Similar to worm-hole switching, but if the channel is blocked, the complete message is buffered at the node. At high network load, it behaves like packet switching.

Latency:

$$t_{vct} = D(t_r + t_s + t_w) + \max(t_s, t_w) \left\lceil \frac{L}{W} \right\rceil$$

INTERCONNECTION NETWORKS

- **A major component of a parallel computer, providing connections among processors and/or memory modules.**
- **Static networks (or direct networks): dedicated links between nodes (point-to-point connections).**
- **Dynamic networks (or indirect networks): network links can form different physical paths from sources to destinations (end-to-end connections).**

- Interconnection Networks
  - Shared-Medium Networks
    - Local Area Networks
      - Contention Bus (Ethernet)
      - Token Bus (Arcnet)
      - Token Ring (FDDI Ring, IBM Token Ring)
    - Backplane Bus (Sun Gigaplane, DEC AlphaServer8X00, SGI PowerPath-2)
  - Direct Networks (Router-Based Networks)
    - Strictly Orthogonal Topologies
      - Mesh
        - 2-D Mesh (Intel Paragon)
        - 3-D Mesh (MIT J-Machine)
      - Torus (k-ary n-cube)
        - 1-D Unidirectional Torus or Ring (KSR first-level ring)
        - 2-D Bidirectional Torus (Intel/CMU iWarp)
        - 3-D Bidirectional Torus (Cray T3D, Cray T3E)
      - Hypercube (Intel iPSC, nCUBE)
    - Other Topologies: Trees, Cube-Connected Cycles, de Bruijn Network, Star Graphs, etc.
  - Indirect Networks (Switch-Based Networks)
    - Regular Topologies
      - Crossbar (Cray X/Y-MP, DEC GIGAswitch, Myrinet)
      - Multistage Interconnection Networks
        - Blocking Networks
          - Unidirectional MIN (NEC Cenju-3, IBM RP3)
          - Bidirectional MIN (IBM SP, TMC CM-5, Meiko CS-2)
        - Nonblocking Networks: Clos Network
    - Irregular Topologies (DEC Autonet, Myrinet, ServerNet)
  - Hybrid Networks
    - Multiple-Backplane Buses (Sun XDBus)
    - Hierarchical Networks (Bridged LANs, KSR)
      - Cluster-Based Networks (Stanford DASH, HP/Convex Exemplar)
    - Other Hypergraph Topologies: Hyperbuses, Hypermeshes, etc.

Figure 1.2. Classification of interconnection networks. (1-D = one-dimensional; 2-D = two-dimensional; 3-D = three-dimensional; CMU = Carnegie Mellon University; DASH = Directory Architecture for Shared-Memory; DEC = Digital Equipment Corp.; FDDI = Fiber Distributed Data Interface; HP = Hewlett-Packard; KSR = Kendall Square Research; MIN = Multistage Interconnection Network; MIT = Massachusetts Institute of Technology; SGI = Silicon Graphics Inc.; TMC = Thinking Machines Corp.)

- **Network control:**
  Generate the necessary control setting on the switches to ensure reliable data routing from source to destination.

- **Control strategies:**
  - **Centralized control:**
    A single network controller takes requests from each input (source) and establishes paths. Easy to use global information to obtain optimal path settings.
  - **Distributed control:**
    Control circuit is associated to each switch/node. Each switch/node uses local information and a routing tag stored in packets.

- **Network design factors:**
  - **Network size:**
    The number of nodes in the network.
  - **Message latency (or network latency):**
    The time elapsed between the time a message is generated at its source node and the time is delivered at its destination node.
  - **Network throughput:**
    The maximum amount of information delivered by the network per time unit.
  - **Scalability:**
    As the network size increases, the network bandwidth should increase proportionally.
  - **Node degree:**
    The number of links incident on a node, denoted as d.

* **Network diameter:**

  The maximum of the shortest path between any two nodes, proportional to network latency.

* **Expandability:**

  The ability to add a node, depending on the number of components and connections required for adding a node.

* **Redundancy (Reliability):**

  The number of different paths between a source and a destination.

* **Bisection width:**

  Cut the network into two halves. The minimum number of links along the cut, denoted as b. It indicates the maximum communication bandwidth.

* **Routing algorithm complexity:**

  Fast or slow. Affects network latency.

- **Routing functions (or interconnection functions)**
  - **Rotation:** $+1$ **mod** N
  - **Shifting:** $+i$ **mod** N
  - **Mesh function (for an** $n \times n$ **mesh)**

$$M_{+1}(x) = (x+1) \textbf{ mod } N$$
$$M_{-1}(x) = (x-1) \textbf{ mod } N$$
$$M_{+n}(x) = (x+n) \textbf{ mod } N$$
$$M_{-n}(x) = (x-n) \textbf{ mod } N$$

  - **Shuffle-exchange:**

    Let $m = \log N$ **and represent a node in binary** $b_{m-1}b_{m-2}\ldots b_1b_0$**.**

    Shuffle function

$$S(b_{m-1}b_{m-2}\ldots b_1b_0) = b_{m-2}b_{m-3}\ldots b_0b_{m-1}$$

    Exchange function

$$E(b_{m-1}b_{m-2}\ldots b_1b_0) = b_{m-1}b_{m-2}\ldots b_1\bar{b_0}$$

$\log N$ **passes of shuffle-exchange function can implement all permutations.**

– **Cube function**

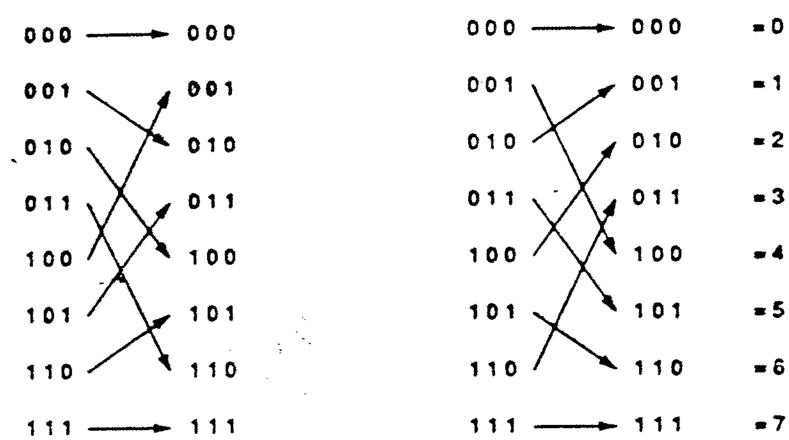
$$C_i(b_{m-1}b_{m-2}\dots b_1b_0) = b_{m-1}b_{m-2}\dots \bar{b_i}\dots b_1b_0$$

for $0 \le i < m$**.**

– **Plus minus** 2^i **(PM2I) function**

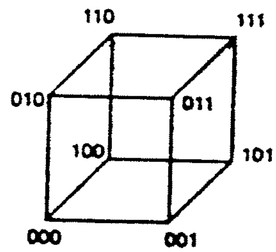
$$PM2_{+i}(x) = (x + 2^i) \textbf{ mod } N$$
$$PM2_{-i}(x) = (x - 2^i) \textbf{ mod } N$$

for $0 \le i < m$**.**



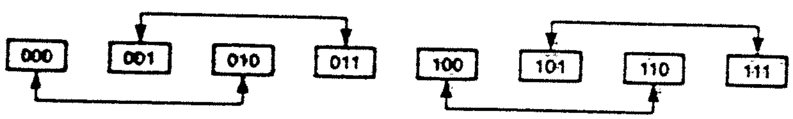
(a) Perfect shuffle (b) Inverse perfect shuffle

Fig 2-14

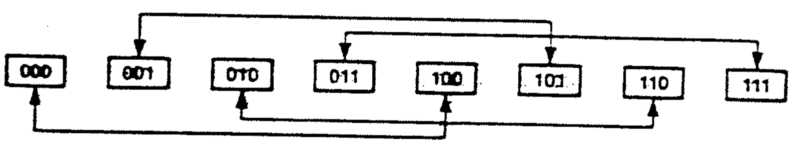


(a) A 3-cube with nodes denoted as $C_2C_1C_0$ in binary

(b) Routing by least significant bit, C_0



(c) Routing by middle bit, C_1



(d) Routing by most significant bit, C_2

Fig 2.15

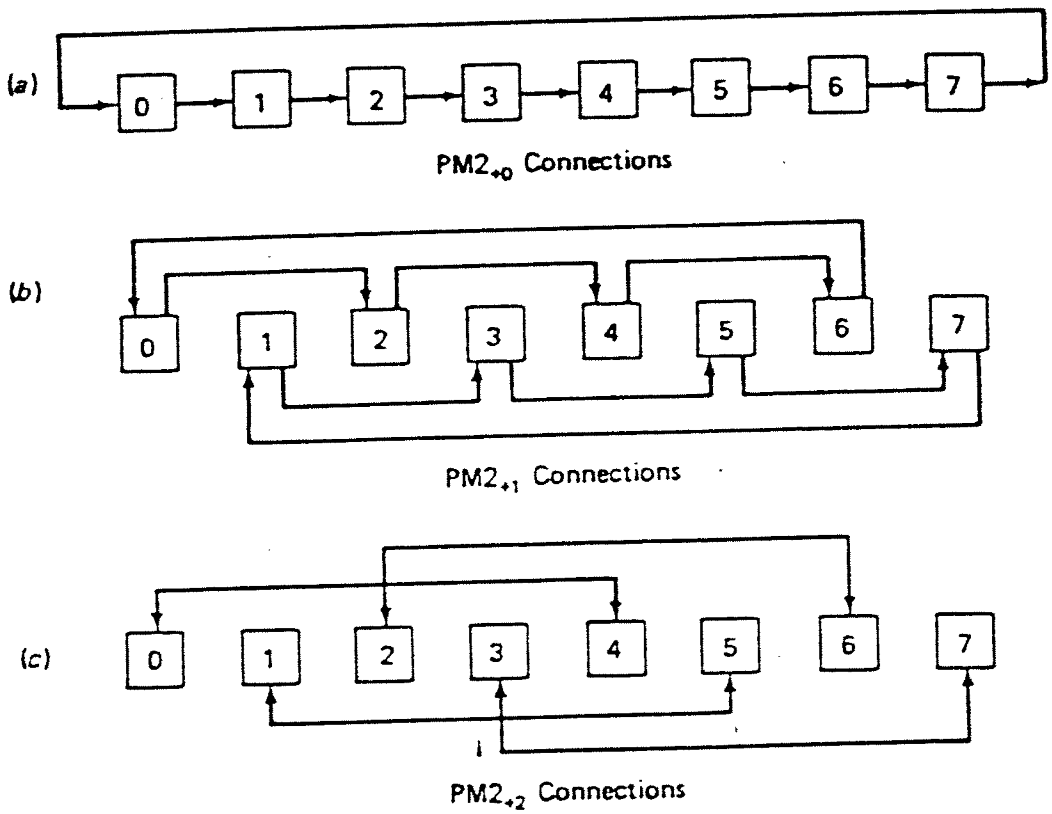


Figure 3.16. PM2I network for $N = 8$ ($PM2_{-i}$ connections have arrows in the opposite directions).

- **Network performance measures**
  - **Data routing capability:**
    blocking, nonblocking, permutation, multicast, etc.
  - **Hardware cost:**
    the number of links, number of switches
  - **Network Latency**
  - **Bandwidth (data rate)**
  - **Scalability:**
    performance increases as the network size increases

- **Typical interconnection networks**
  - **Static networks**

    Fixed links between nodes, suitable to the applications with communication patterns match the structure of the network.

    * **Ring based networks**
      - **Linear array**

        Degree $d = 2$

        Diameter $D = N - 1$

        Bisection $b = 1$

        Different from a bus.

      - **Ring**

        Degree $d = 2$

        Diameter $D = \lfloor \frac{N}{2} \rfloor$

        Bisection $b = 2$

      - **Chordal ring**

N: **number of nodes, even**

W: **chordal length, odd**

Every odd-numbered node p **(**$p = 1, 3, \ldots, N-1$**) is connected to** $(p + W)$ **mod** N **(an even-numbered node).**

Degree $d = 3$

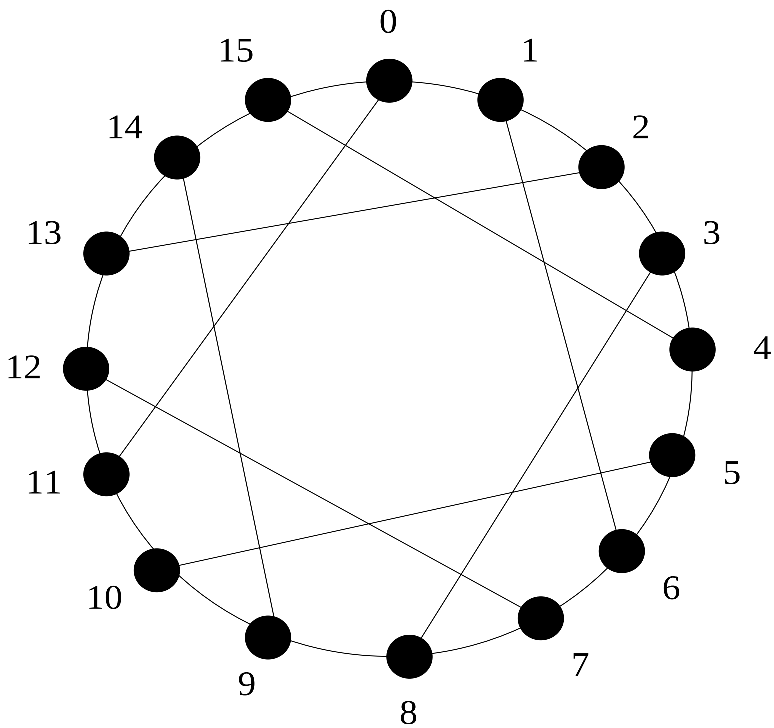
Diameter $D = O(\sqrt{N})$

Bisection $b = 6$

Basic routing algorithm:

Follow ring edge and chordal alternating path to the nearby area, then follow edges within a chordal distance.

- **Further generalization: two chordals.**



A 16 node chordal ring

- **Completely connected**

  Degree $d = N - 1$

  Diameter $D = 1$**.**

- **Barrel shifter**

  $N = 2^n$

  Node i **is connected to node** j **if** $|j - i| = 2^r$ **for** $r = 0, 1, \ldots, n - 1.$

  Degree $d = 2n - 1$

  Diameter $D = n/2$

– **Tree based networks**

- **Star**

  Degree $d = N - 1$

  Diameter $D = 2$

- **Tree**

  Binary tree: $N = 2^k - 1$ **nodes**

  Degree $d = 3$

  Diameter $D = 2(k - 1)$, $O(\log N)$

  Constant degree, but heavy traffic at root node.

- **Fat tree**

  Thinking machines Connection machine CM-5 uses this network.

  Basic idea: wider channels towards the root to release the bottleneck but not constant degree any more.

- **Mesh based networks**
  - **k-dimensional mesh**

    $N = n^k$ **nodes,** n **nodes in each dimension and each node has two neighbors in each dimension**

    Degree $d = 2k$

    Diameter $D = k(n-1)$
  - **Illiac IV network**

    Two dimensions, 64 nodes, $D = n - 1$
  - **Torus**

    Similar to mesh, but symmetric

    $D = 2\lfloor \frac{n}{2} \rfloor$**.**

    In general, for k**-dimensional,**

    $D = K\lfloor \frac{n}{2} \rfloor$**.**
  - **Systolic arrays**

    Pipelined array architecture for implementing fixed algorithm. Two dimension, but

the degree is not necessarily 4, can be larger. Matches the communication pattern of the algorithm.

- **Cube based networks**
  - **Hypercube**
    n-cube architecture with $N = 2^n$ nodes.
    * **Geometrical definition: N nodes on the corner of n "cube" in n-space**
    * **Recursive definition: Form a hypercube of dimension n by taking two hypercubes of dimension $n-1$ and directly connecting corresponding nodes.**
    * **Interconnection function:**
      Cube function, connect the nodes iff they have only one-bit difference.
  - **Degree $d = \log N$**
    Diameter $D = \log N$
  - **Easy routing: only need to look at bit i of the destination node at step i.**
  - **Drawback: variable degree, poor expandability.**

- **Cube-connected cycles (CCCs)**

  A hierarchical network.

  Replace each node in an n-cube with a small ring with n-nodes.

  $N = 2^n \times n$ **nodes**

  Constant degree for any n: $d = 3$

  Slightly shorter diameter $D = 2n - 1 + \lfloor \frac{n}{2} \rfloor = O(\log N)$**.**

- **Even poorer expandability**

- **k-ary n-cube networks**

  Radix k ($k = 2$: binary hypercube)

  Each node represented as $a_{n-1}a_{n-2}\ldots a_0$ **with** $0 \leq a_i \leq k-1$ **for** $i = 0, 1, \ldots, n-1$**.**

  n **dimensions, each dimension has** k **nodes, connected as a cycle.**

  Each dimension connected to "plus minus 1" nodes

  e.g. $k = 4$, $n = 3$

$N = k^n$ **nodes,** $k = N^{1/n}$**,** $n = \log_k N$**.**

Degree $d = 2n$

Diameter $D = n\lfloor \frac{k}{2} \rfloor$**.**

- **Summary of static networks**

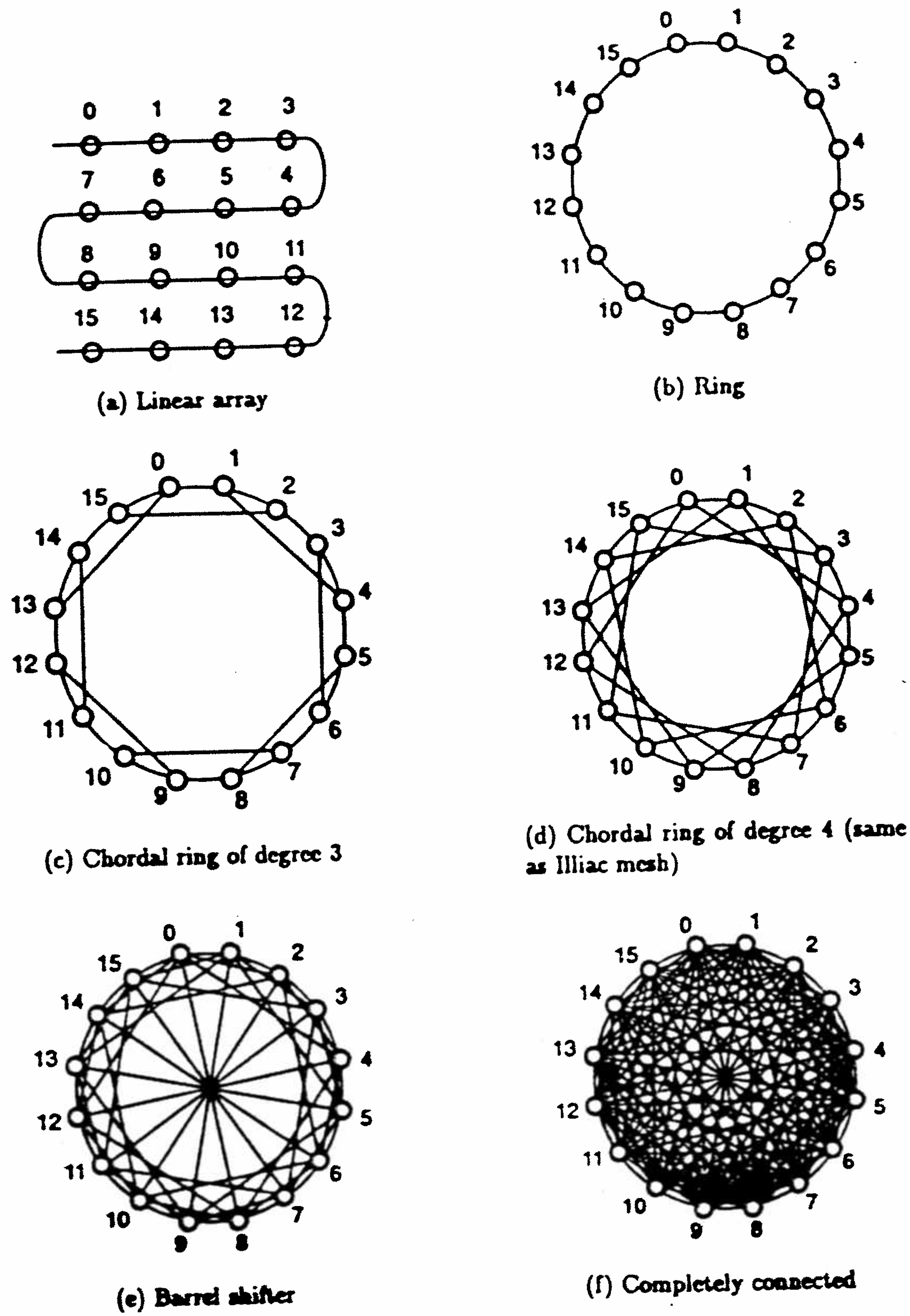
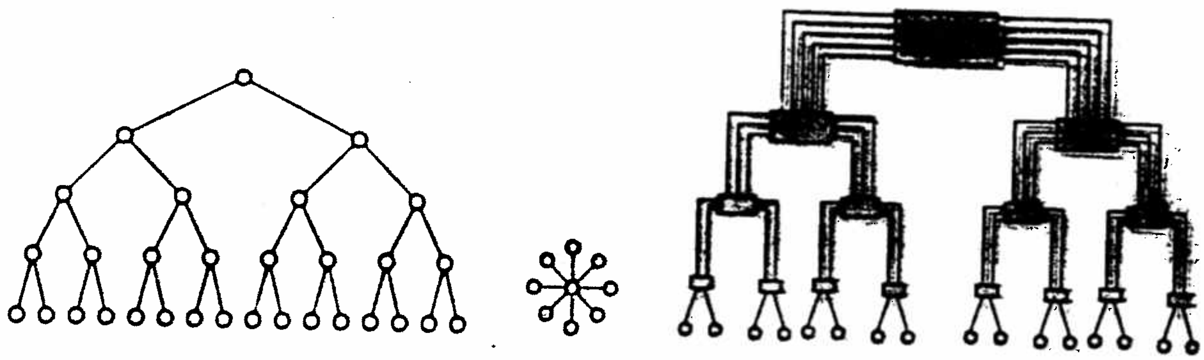


Figure 2.16 Linear array, ring, chordal rings of degrees 3 and 4, barrel shifter, and completely connected network.



(a) Binary tree (b) Star (c) Binary fat tree

Figure 2.17 Tree, star, and fat tree.

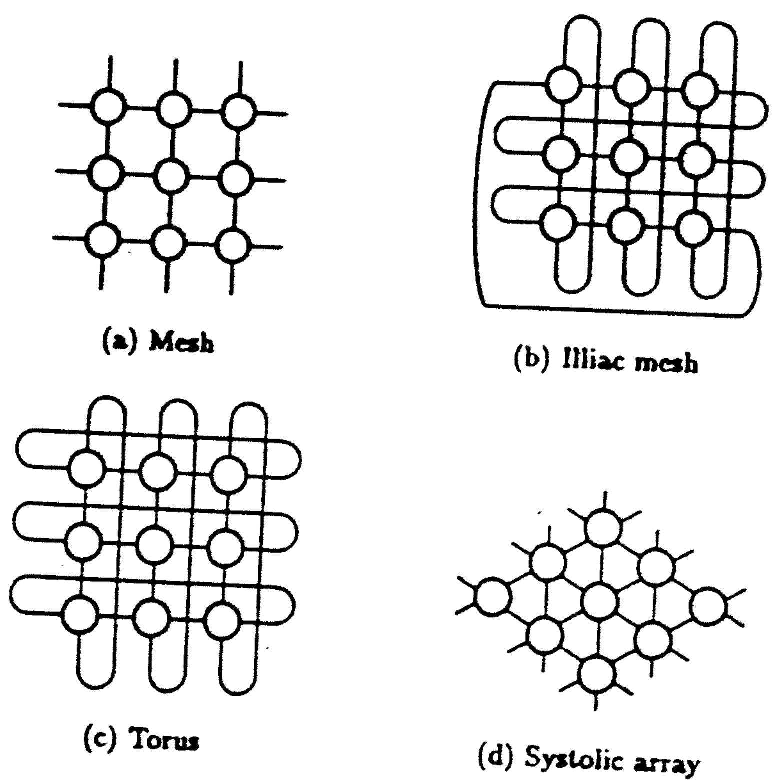
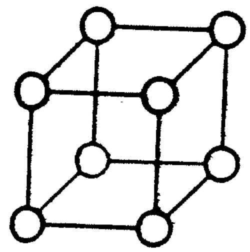
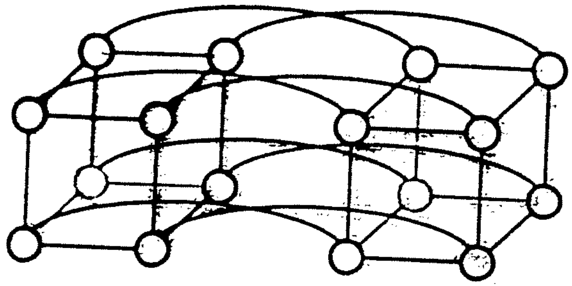


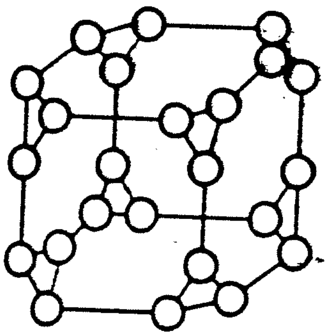
Figure 2.18 Mesh, Illiac mesh, torus, and systolic array.



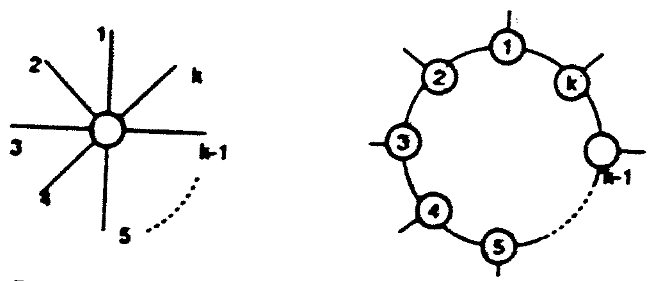
(a) 3-cube



(b) A 4-cube formed by interconnecting two 3-cubes



(c) 3-cube-connected cycles



(d) Replacing each node of a k-cube by a ring (cycle) of k nodes to form the k-cube-connected cycles

Figure 2.19 Hypercubes and cube-connected cycles.

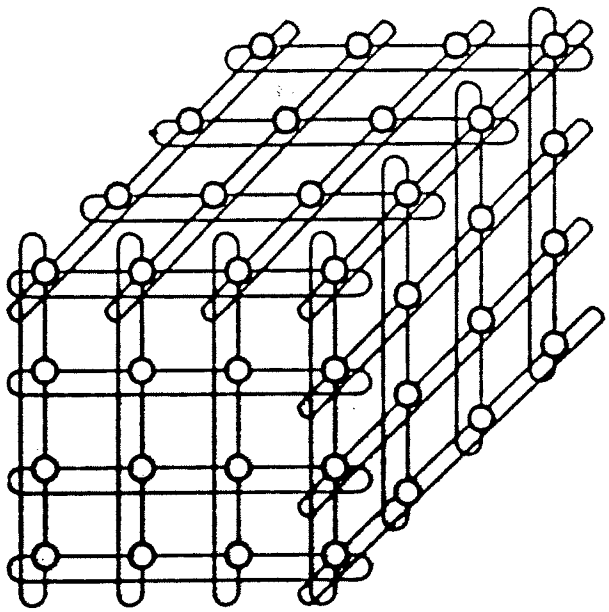


Figure 2.20 The k-ary n-cube network shown with k = 4 and n = 3; hidden nodes or connections are not shown.

Table 2.2 Summary of Static Network Characteristics

Network type	Node degree, d	Network diameter, D	No. of links, l	Bisection width, B	Symmetry	Remarks on network size
Linear Array	2	$N-1$	$N-1$	1	No	N nodes
Ring	2	$\lfloor N/2 \rfloor$	N	2	Yes	N nodes
Completely Connected	$N-1$	1	$N(N-1)/2$	$(N/2)^2$	Yes	N nodes
Binary Tree	3	$2(h-1)$	$N-1$	1	No	Tree height $h = \lceil \log_2 N \rceil$
Star	$N-1$	2	$N-1$	$\lfloor N/2 \rfloor$	No	N nodes
2D-Mesh	4	$2(r-1)$	$2N-2r$	r	No	$r \times r$ mesh where $r = \sqrt{N}$
Illiac Mesh	4	$r-1$	$2N$	$2r$	No	Equivalent to a chordal ring of $r = \sqrt{N}$
2D-Torus	4	$2\lfloor r/2 \rfloor$	$2N$	$2r$	Yes	$r \times r$ torus where $r = \sqrt{N}$
Hypercube	n	n	$nN/2$	$N/2$	Yes	N nodes, $n = \log_2 N$ (dimension)
CCC	3	$2k-1+\lfloor k/2 \rfloor$	$3N/2$	$N/(2k)$	Yes	$N = k \times 2^k$ nodes with a cycle length $k \geq 3$
k-ary n-cube	$2n$	$n\lfloor k/2 \rfloor$	nN	$2k^{n-1}$	Yes	$N = k^n$ nodes

- **Dynamic interconnection networks**
  - **Implement all communication patterns, suitable to general purpose applications.**
  - **Components: switches and sharable links**
  - **Dynamically change the path settings**
  - **Cost of dynamic network: switches and links, usually in terms of crosspoints.**
  - **Performance measures: bandwidth, latency, communication patterns supported.**

- **Types of dynamic networks (in the increasing order of cost and performance):**
  Buses – multistage interconnection networks (MINs) – Crossbars
  - **Buses**
    Time sharing, low cost, very limited bandwidth.
    One transaction at a time. Only one pair of nodes can use the bus. Not scalable, vulnerable to bus controller failures.
  - **Multistage network consists of switch modules and links**
    Switch module: $a \times b$ **switch module with** a **input and** b **output.**
    Crosspoints: ab
    One-to-one connection switch
    One-to-many connection switch
    Legitimate states

Group switches into stages. Connect stages by certain interconnection functions.

- **Crossbar**

  1 stage, the most powerful connecting capability, $O(N^2)$ switches

- **Generalized cube network**

  $N \times N$ **network**

  $N/2$ **switches in each stage**

  $n = \log N$ **stages, numbered from** $n-1$ **to 0**

  Interconnection function c_i **(cube) function for stage** i

  Setting switch to swap at stage i **realizes** c_i **function**

  Routing algorithm (distributed)

Source $S = S_{n-1}S_{n-2}\ldots S_1S_0$

Destination $D = D_{n-1}D_{n-2}\ldots D_1D_0$

The switch at stage i **in the path from** S **to** D **must be set to swap if** $D_i \neq S_i$ **and set to straight if** $D_i = S_i$**.**

Unique path from S **to** D**.**

Routing example.

∗ **Data manipulator network**

$N \times N$ **network**

Each stage has N **switching elements**

Each switching element accepts one from three input links and outputs one from three output links (implemented by DEMUX and MUX)

Interconnection function of stage i**:**

- $PM2_{+i}$
- $PM2_{-i}$

· **Straight connection**

Control signals:

· **S - straight**

· **U - up** (-2^i)

· **D - down** $(+2^i)$

Routing:

From source S to destination D. Compute link sum $(D - S)$ **mod** N **and decompose it into the sum of power of 2**

$*$ **Omega network**

$N/2$ 2×2 **switches at each stage**

$\log N$ **stages**

Each stage has identical interconnection function: shuffle exchange

Routing:

Controlled by the address of the destination node

At stage i, if $D_i = 0$ go to upper output of the switch, if $D_i = 1$ go to lower output of the switch.

The number of permutations an $N \times N$ network can realize: $N^{N/2}$.

∗ **Baseline network (general structure of blocking network)**

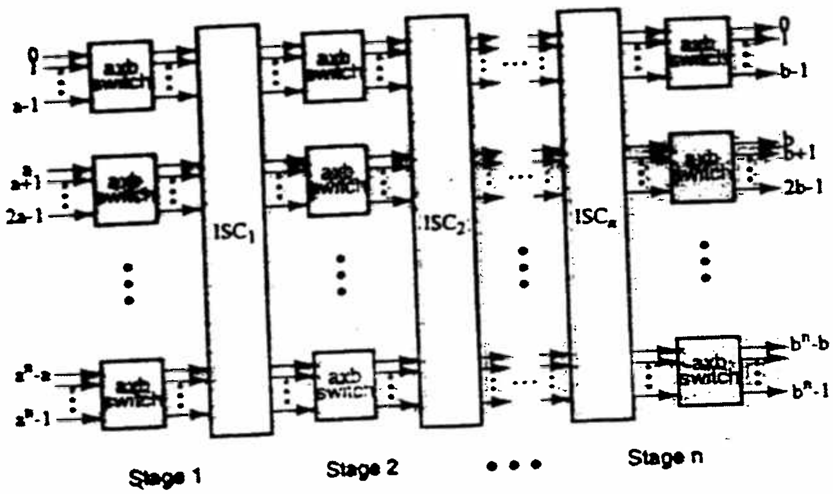
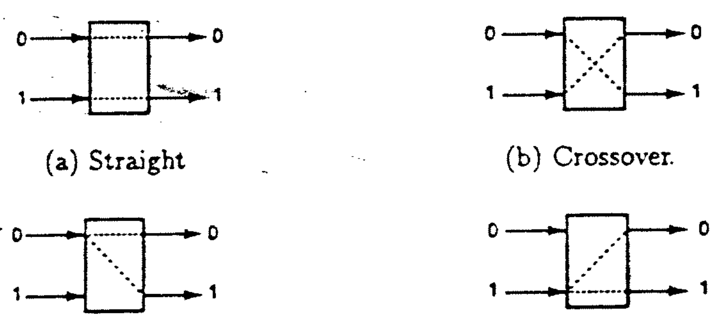


Figure 2.23 A generalized structure of a multistage interconnection network (MIN) built with a x b switch modules and interstage connection patterns $ISC_1, ISC_2, \ldots, ISC_n$.

Table 2.3 Switch Modules and Legitimate States

Module Size	Legitimate States	Permutation Connections
2 × 2	4	2
4 × 4	256	24
8 × 8	16,777,216	40,320
$n \times n$	n^n	$n!$



(a) Straight (b) Crossover. (c) Upper broadcast (d) Lower broadcast

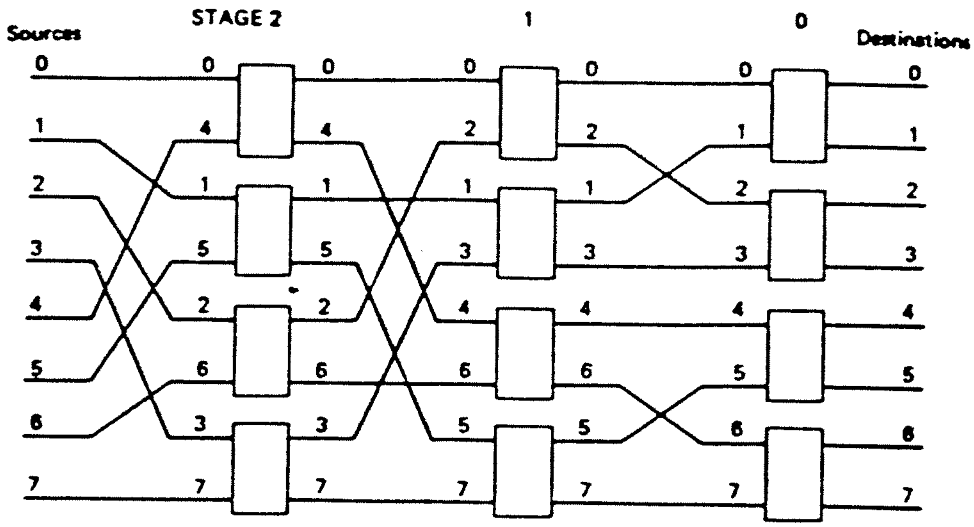


Figure 3.17. Generalized Cube network for $N = 8$.

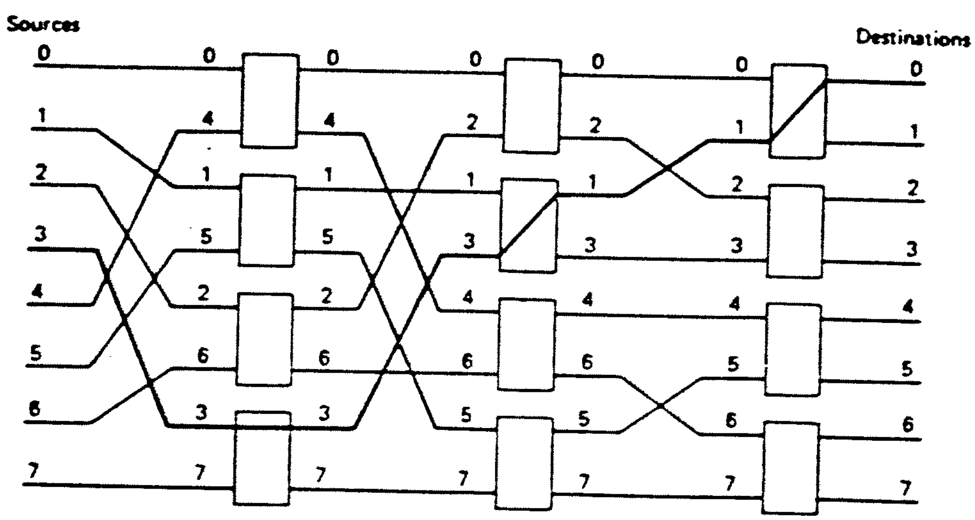


Figure 3.19. Path from source 3 to destination 0 in the Generalized Cube network for $N = 8$.

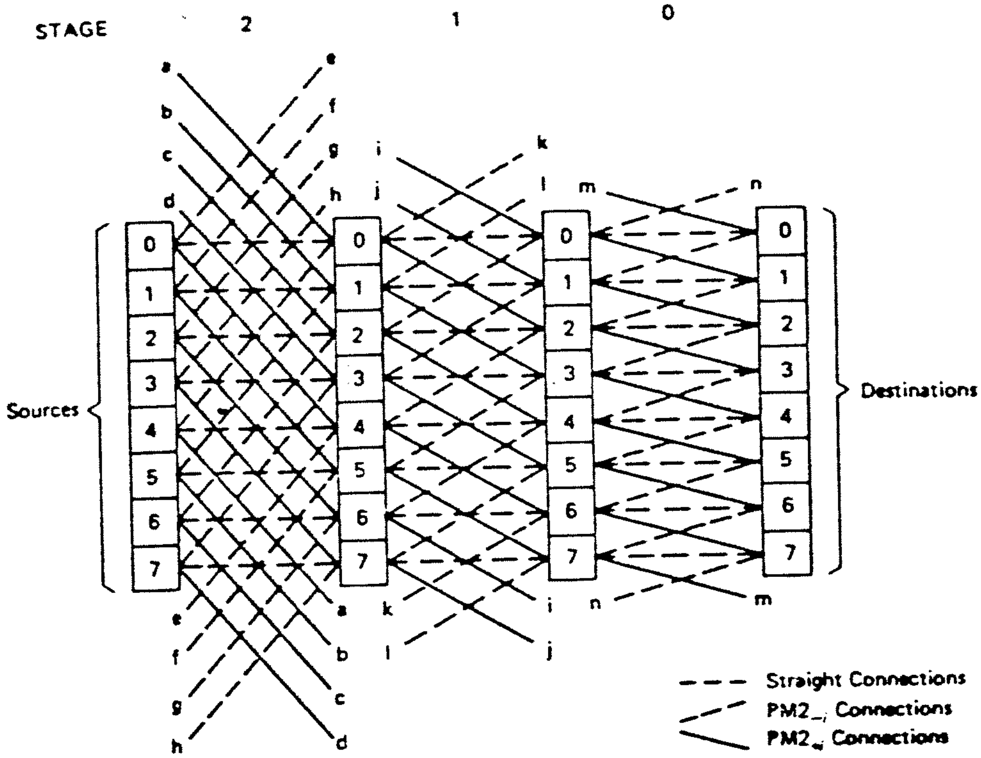


Figure 3.21. The Data Manipulator network for $N = 8$.

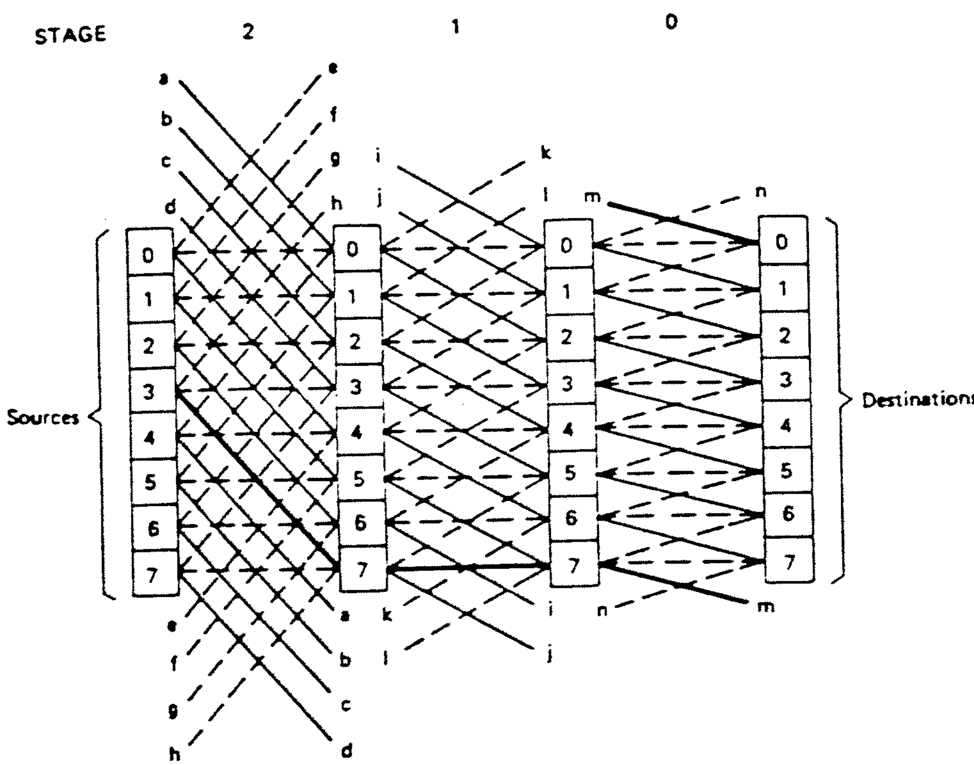


Figure 3.23. Path (bold line) from source 3 to destination 0 in the Data Manipulator for $N = 8$.

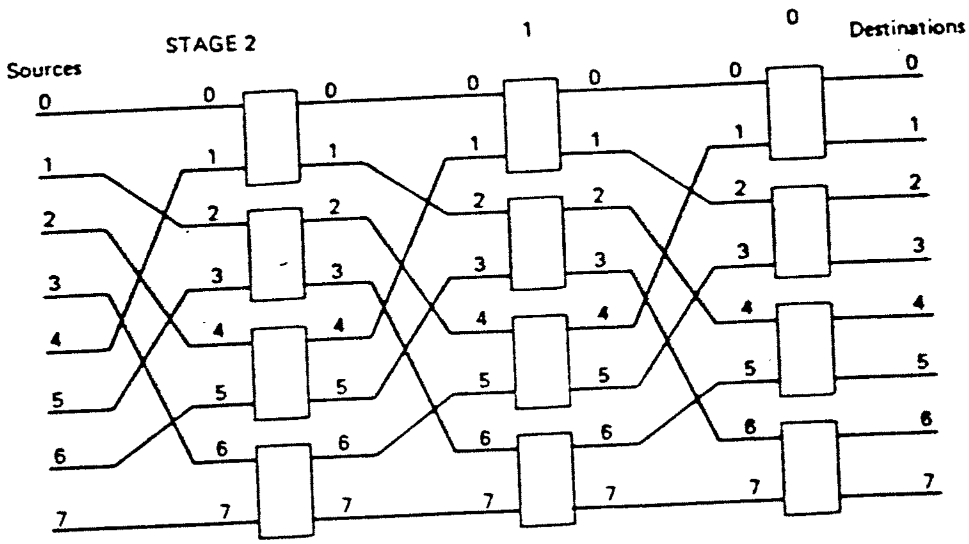
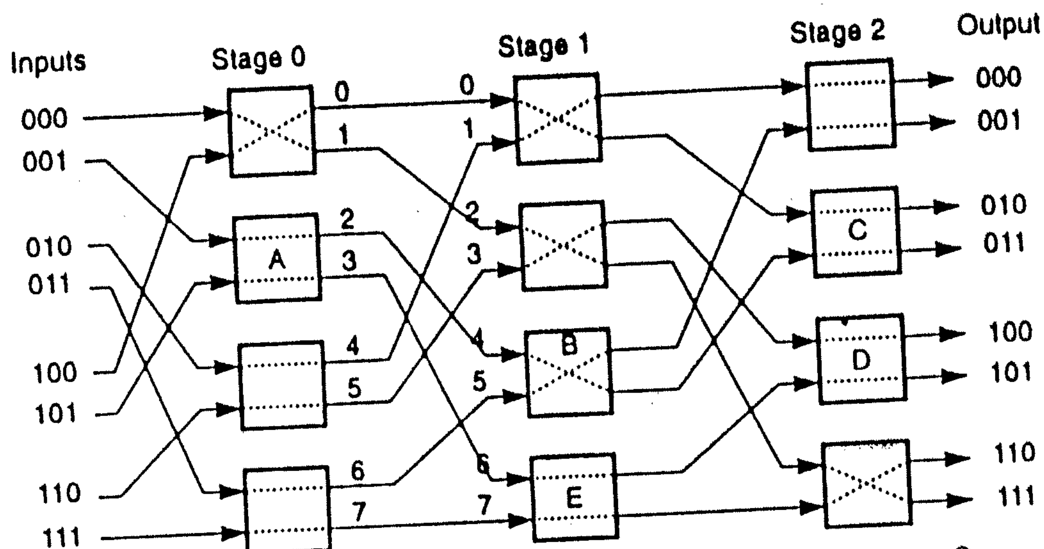
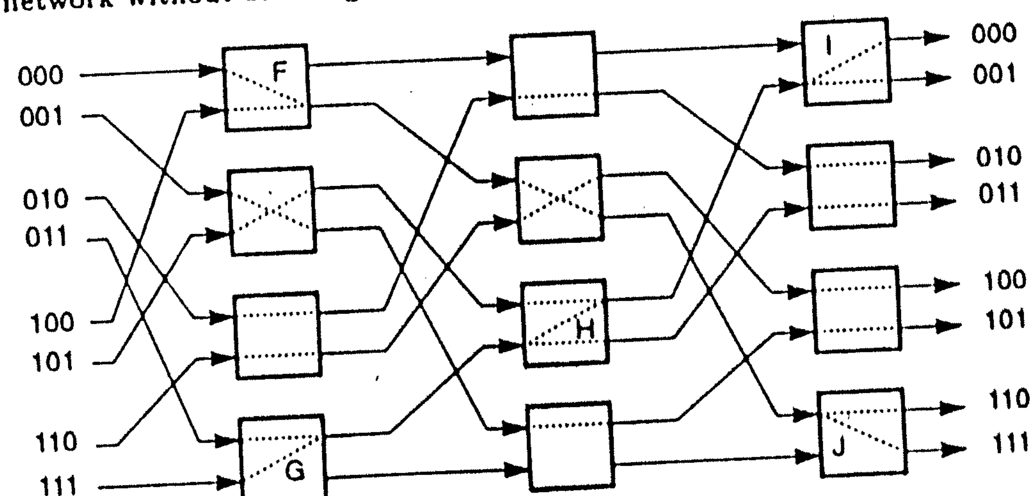


Figure 3.25. Omega network for $N = 8$.

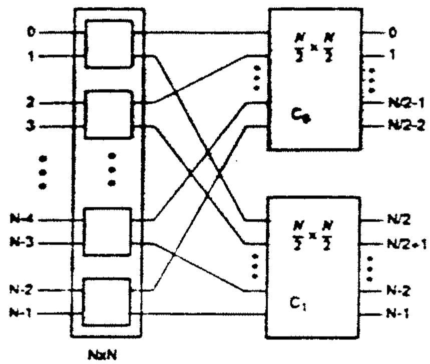


(a) Permutation $\pi_1 = (0,7,6,4,2)(1,3)(5)$ implemented on an Omega network without blocking

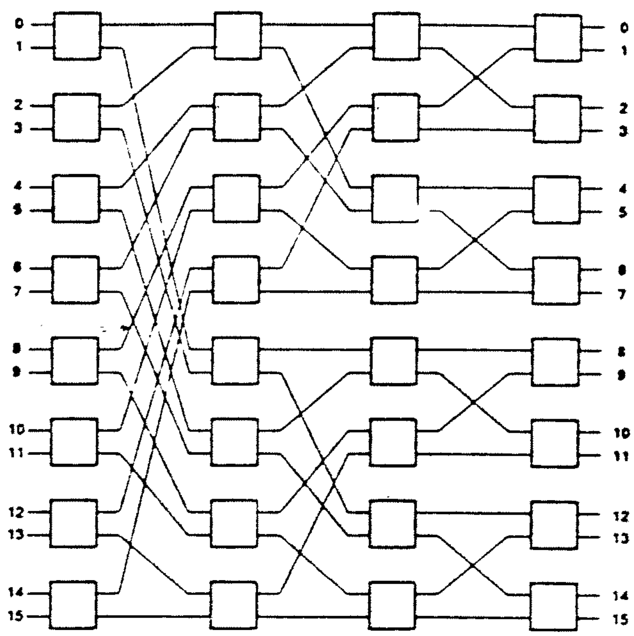


(b) Permutation $\pi_2 = (0,6,4,7,3)(1,5)(2)$ blocked at switches marked F, G, and H

Figure 7.8 Two switch settings of an 8×8 Omega network built with 2×2 switch



(a) Recursive construction



(b) A 16 × 16 Baseline network

Figure 2.25 **Recursive construction of a Baseline network.** (Courtesy of Wu and Feng; reprinted with permission from *IEEE Trans. Computers, August 1980*)

- **Clos network (also called $v(m, n, r)$ network)**
  - **Network structure**

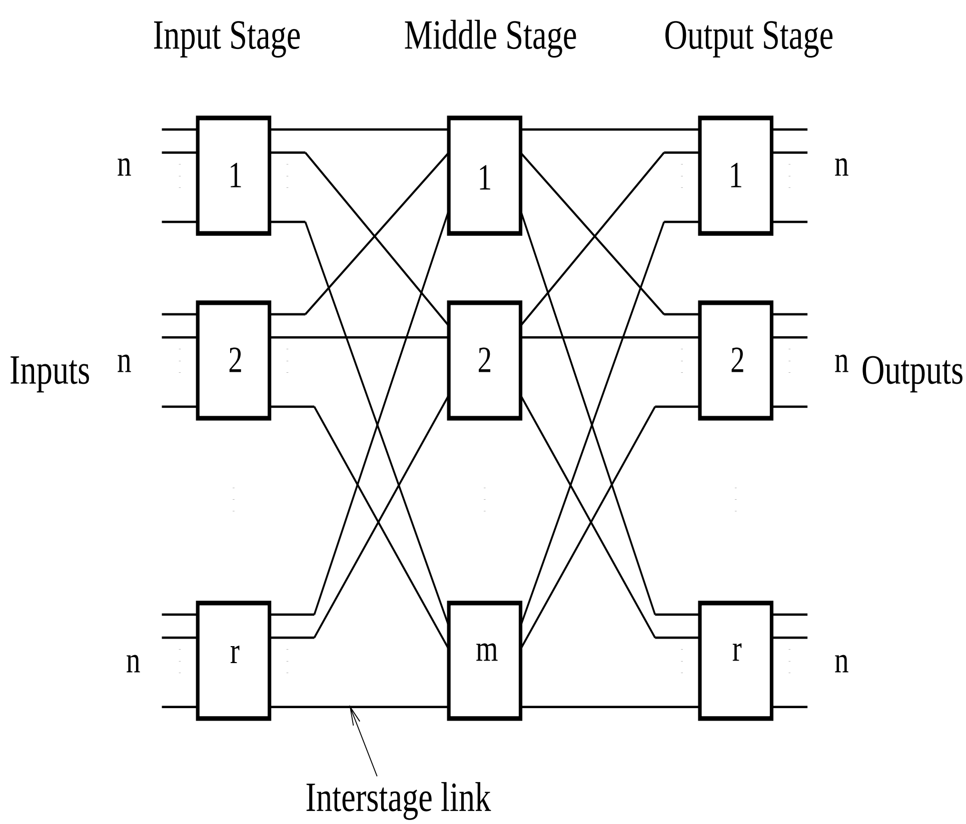
    Three stages of switches

    Input stage: r $n \times m$ **switches**

    Middle stage: m $r \times r$ **switches**

    Output stage: r $m \times n$ **switches**

    $N = nr$ **inputs/outputs**



– **Rearrangeable permutation network**

Condition: $m \geq n$

Rearrangeable for permutation: can satisfy any new connection request from an idle input to an idle output, but sometimes it is necessary to interrupt and rearrange the existing connections in the network.

– **Nonblocking permutation network**

Condition: $m \geq 2n - 1$

Nonblocking for permutation: can satisfy any new connection request from an idle input to an idle output and the rearrangement is never required.

– **Multicast network**

Condition: $m = O\left(n \frac{\log r}{\log \log r}\right)$ **for both nonblocking and rearrangeable multicast.**

– **Proof of rearrangeable permutation condition $m \geq n$**

Basic combinatorial theorem:

Hall's Theorem: Let A be any finite set, and let $A_1, A_2, \ldots, A_r$ be any r subsets of A. A necessary and sufficient condition that there exist a set of distinct representatives $a_1, a_2, \ldots, a_r$ of $A_1, A_2, \ldots, A_r$, i.e. elements $a_1, a_2, \ldots, a_r$ of A such that

$$a_i \in A_i, \quad i = 1, 2, \ldots, r$$
$$a_i \neq a_j \ \ for \ \ j \neq i$$

is that for each k in the range $1 \leq k \leq r$ the union of any k of the sets $A_1, A_2, \ldots, A_r$ have at least k elements.

Proof.

Suppose

inputs are $1, 2 \ldots, N$

outputs are $1, 2, \ldots, N$

Input switches are $I_1, I_2, \ldots, I_r$

Output switches are $O_1, O_2, \ldots, O_r$

Consider a permutation

$$\{i \to \pi(i), i = 1, 2, \ldots, N\}$$

Let

$$K = \{1, 2, \ldots, r\}$$

For any $K_i \subseteq K$**,**

$$K_i = \{j : \pi(l) \in O_j, l \in I_i\}$$

Consider any k **input switches**

$$I_{i(1)}, I_{i(2)}, \ldots, I_{i(k)}$$

corresponding to

$$K_{i(1)}, K_{i(2)}, \ldots, K_{i(k)}$$

$$K_{i(j)} \subseteq K, 1 \leq j \leq k$$

Consider

$$T = \cup_{j=1}^{k} K_{i(j)}$$

Let $|T| = t$**.**

Note that

$$| \cup_{j=1}^{k} I_{i(j)}| = k \times n$$

This is because that each $I_{i(j)}$ **has** n **distinct inputs, and theses** kn **inputs are connected to** t **output switches with a total of** tn **outputs. Therefore,**

$$tn \geq kn$$

Then we have $t \geq k$**.**

That is,

$$T = \cup_{j=1}^{k} K_{i(j)}$$

has at least k **elements. Then by Hall's theorem, there exists a set of distinct represen-**

tatives

$$k(i) \in K_i, i = 1, 2, \ldots, r$$

$$k(i) \neq k(j)$$

Thus we have a mapping from each input switch to a distinct output switch:

$$I_i \to K(i)$$

Since all these connections are from different input switches to different output switches, we can direct all connections to a single middle switch.

The remaining network becomes
a $v(m-1, n-1, r)$ network.
Note that $v(1, 1, r)$ is a permutation network.
By induction on n, we know that a network with $m = n$ can realize all permutations.

– **Nonblocking permutation network**

$m \geq 2n - 1$**.**

Consider connecting an input from input switch i **to an output of output switch** j**. Note that at most** $n - 1$ **inputs on input switch** i **can be busy and at most** $n - 1$ **outputs on output switch** j **can be busy. So we need one more middle switch to make the new connection. Thus, the number of middle switches needed for nonblocking is**

$$(n - 1) + (n - 1) + 1 = 2n - 1.$$

- **Number of crosspoints**

$$\#cp = 2n \times m \times r + r^2 \times m$$

  When $m = n = r$**,** $\#cp = 3N^{3/2}$

- **Generalization to** $2k+1$ **stage for any** $k \geq 1$**.**
  Replacing each $r \times r$ **middle switch by an** $r \times r$ **Clos network.**

  Crosspoints:

$$\#cp = O(N^{1+1/k})$$

  for $2k+1$ **stage network.**

- **A special type of Clos network: Benes network. Set** $m = n = 2$ **in Clos network. Recursive construction until all switches become** 2×2 **switches.**

  $2 \log N - 1$ **stages**

  $\#cp = O(N \log N)$

  A $O(N \log N)$ **permutation network.**

- **Summary of dynamic networks**
  - **Buses**

    $\#cp = O(N)$

  - **Multistage networks**
    - $\log N$ **stage networks**

      $\#cp = O(N \log N)$

      Most are blocking networks.

    - **Constant stage networks**

      $\#cp = O(N \cdot N^{1/k})$

      Rearrangeable networks

      Nonblocking networks

  - **Crossbars**

    $\#cp = N^2$