Twinflows: Markov Chain Flow Decomposition for Tandem Queues with Blocking

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Abstract — **In this short paper it is reported that the circulatory structure of two tandem finite buffer queues using Markovian statistics, indeed can be decomposed in a simple and elegant manner. The interesting point concerning any possible decomposition of flows is that one might reasonably think that for any non-product form network, the circulatory structure would consist of some arbitrary pattern of flows. Thus the existence of a simple, structured, decomposition is somewhat surprising. It may in fact lead to new computational algorithms for Markov chains.**

I. Introduction

Markov chains have been a mathematically elegant means of stochastic modeling for almost a century. It is well known that for any arbitrary Markov chain, the equilibrium state probabilities can be computed by solving the associated set of linear global balance equations. Unfortunately numerical solution techniques for N state arbitrary Markov chains can involve a computational complexity of O(N sup 3), allowing only arbitrary Markov chains of modest size to be exactly solved. However since the original work of Jackson in 1957 [1], and later Gordon and Newell in 1967 [2], it has been known that there is a class of both open and closed queuing networks with a tractable analytic solution of the product form type. That is, for this product form class of queueing networks any state equilibrium probability is a product of system parameters and a reference probability.

In 1984 Lazar and Robertazzi [3] showed that the circulatory structure of product form network Markov chains can be decomposed into an aggregation of simpler (often cyclic) circulations. However product form networks are a limited class of queuing networks. Other types of queuing networks have been known collectively as non-product form networks.

II. Main Result

Software has been developed that demonstrates that the flow of probability flux in the state transition diagram of a two tandem finite buffer queue can be exactly decomposed as in Figure 1. That is, the flow can be decomposed into "0" and "1" (twin) cyclic flows (some of which may be negative). Interestingly, related product form circulations consists solely of "0" flows.

This non-product form circulatory structure may be of interest for developing alternate computational algorithms for equilibrium state probabilities. It should be noted that since the algebraic topology of non-product form state transition diagrams are so different from one another, such solutions for each protocol model needs to be customized.

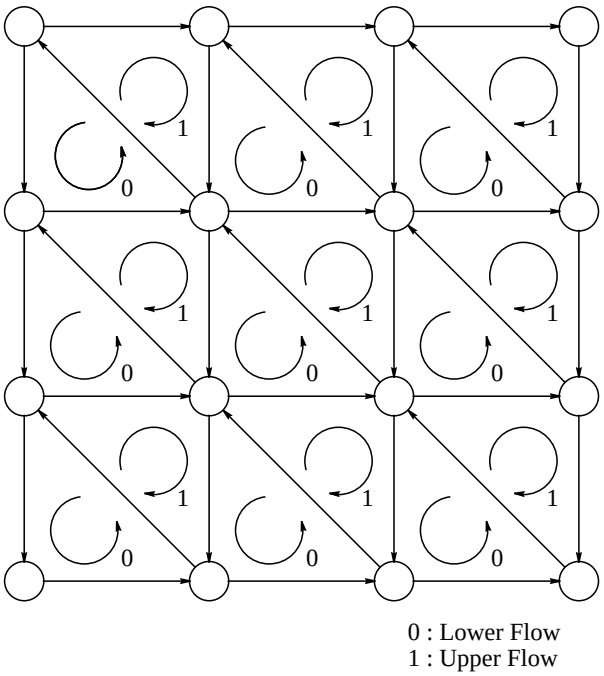

Figure 1

III. Conclusion

This initial work indicates that non-product form queueing network Markov chains can have a surprisingly simple flow structure. This may enable the development of alternate numerical algorithms for queueing performance metrics. Moreover it is a surprising positive result in indicating the possible existence of patterned algebraic topological structure in many Markov chains of practical interest.

References

[1] J.R. Jackson, "Networks of Waiting Lines," Operations Research, vol. 5, 1957, pp. 518-521.

[2] W.J. Gordan and G.F. Newell, "Closed Queueing Systems with Exponential Servers," Operations Research, vol. 15, 1967, pp. 254-265.

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