Divisible Load Scheduling and Markov Chain Models

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Abstract

In this paper the equivalence between various divisible load-scheduling policies and continuous time Markov chains is demonstrated. This provides a basic unification of both data parallel divisible load scheduling and Markov chain models for the first time in 16 years of research. Such equivalence is demonstrated for divisible scheduling on linear daisy chains and single and two level tree networks.

Key words: Divisible loads, scheduling, divisible jobs, Markov chains, linear networks, tree networks

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1 Introduction

Divisible load scheduling theory (DLT) involves the study of the optimal distribution of partitionable loads among a number of processors and links [1,2,3,4]. A partitionable data parallel load is one that can be arbitrarily distributed among the processors and links in a system. Thus there are no precedence relations among the data. Applications include grid computing, parallel and distributed processor network scheduling, data intensive computing and meta-computing. The approach is particularly suited to the processing of very large data files as in signal processing, Kalman filtering, image processing, experimental data processing, multimedia and computer utility applications.

There has been an increasing amount of study on divisible load scheduling theory since the work of Cheng and Robertazzi [5] in 1988. Most of these studies develop an efficient allocation of load to processors over a network by forcing the processors to all stop processing at the same time. Intuitively, this is because the solution could be improved by transferring load if some processors were idle while others are still busy [6]. Optimal allocation of loads for network topologies including linear daisy chains, bus networks and tree networks using a set of recursive equations were presented in [5][7][8] respectively. For complex networks, the concept of equivalent networks was presented in [9]. There have been further studies in terms of load distribution policies for two and three dimensional meshes [10] and hypercubes [11]. In [12] the concept of time varying processor speed and link speed are introduced. In [13] the integration of monetary cost optimization and divisible load theory is presented. Scheduling policy research includes independent task scheduling [14,15], multi-installment sequential scheduling [16], multi-round algorithms [17], fixed communication charges [18], detailed parameterization and solution reporting time optimization [19] and combinatorial optimization [20]. An important reason for using divisible load scheduling theory is its tractability, flexibility and realism for a large class of data intensive, data parallel, computational problems.

In this paper equivalent continuous time Markov chain models [21,22,23] for various network topologies and load scheduling policies currently modeled by divisible load theory are introduced. Our initial motivation for introducing this unification between divisible load theory and Markov chain models is that they have a number of commonalities between them. In their basic form the two theories are linear ones. That is, they can be solved in theory by solving the associated linear set of equations. Other common features include a schematic language, recursive or linear equation solutions, the concept of equivalent networks, the possibility for time varying modeling and solutions for infinite size homogeneous networks [24].

In fact we can show many, though not all, optimal divisible load schedules for various network topologies have Markov chain analogs. This helps to explain the similarity between queueing theory and divisible load theory. On the other hand this equivalence is somewhat surprising since divisible load theory is deterministic while Markov chain models are stochastic. This new equivalence provides a novel and apparently powerful modeling tool.

This paper presents examples of this equivalence. While most of the Markov chains are one dimensional in topology, the labeling of transitions is different from the usual practice in queueing theory.

The remainder of this paper is organized as follows. In section 2, basic model description and notation and definitions used through out this paper are presented. Sections 3, 4 and 5 discuss models for linear daisy chains, single level tree and two level tree networks, respectively. In section 6 some open problems are presented as a guide for future work. Finally, the conclusion is contained in section 7.

2 Model Description

In this section, some assumptions for scheduling in divisible load theory are described along with some notation and definitions. As mentioned earlier, the network topologies discussed in detail in this paper include linear daisy chains, and single and two level tree networks. The models discussed in this paper account also for both homogeneous and heterogeneous processing and link speeds and various load scheduling policies.

As mentioned earlier, it will be assumed that the total data parallel processing load is arbitrarily divisible into fractions of loads to be assigned to each processor over a network. The root processor where the total processing load originates, keeps some processing load for itself and sends out the rest of the load to the remaining processors over the network. There are different scenarios for the processors, depending whether or not they can compute and communicate at the same time. In general, we will consider two cases: with front end processors and without front end processors. In the case of processors with front end processors, it is assumed that some of the processors in the network are equipped with front ends so that they are able to compute their own load fraction and communicate (if necessary) simultaneously. In the case of networks without front end processors, it is assumed that none of the processors are equipped with front ends and the processors can only compute or communicate at one time. It is assumed that solution reporting time (back to the load originating node) is negligible compared to load distribution time and so is neglected. However, solution reporting time can be naturally modeled for divisible loads when necessary.

2.1 Notations and Definitions:

In this paper the following notations and their definitions will be used.

α_i: The fraction of load that is assigned to processor i by the load originating processor.

ω_i: A constant that is inversely proportional to the computation speed of processor i in the network.

z_i: A constant that is inversely proportional to the speed of link i in the network.

T_{cp}: Computation intensity constant. This is the time that it takes the i^{th} processor to process the entire load when $\omega_i = 1$. The entire load can be processed on the i^{th} processor in time $\omega_i T_{cp}$.

T_{cm}: Communication intensity constant. This is the time that it takes to transmit the entire processing load over a link when $z_i = 1$. The entire load can be transmitted over the i^{th} link in time $z_i T_{cm}$.

T_i: The total time that elapses between the beginning of the scheduling process at $t = 0$ and the time when processor i completes its computation, $i = 0, 1, ..., n$. This includes, in addition to computation time, communicating time and idle time.

T_f: processing finish time of total processing load, assuming load is delivered to originator processor by $t = 0$. Naturally,

$$T_f = \max_i T_i.$$

In all of the sections the same definitions are used for α_i, ω_i, z_i, T_{cp}, T_{cm}, T_i and T_f unless otherwise stated. Another convention that is followed in is that the load originating at the root processor is assumed to be normalized to be a unit load.

3 Linear Daisy Chain Networks

Consider a linear daisy chain network consisting of $N+1$ processors connected via N communication links as shown in Fig. 1. The root processor P_0, where the load originates keeps its own share of load α_0 and communicates the remaining load (1-α_0) to its immediate successor P_1. Similarly, the processor P_1 keeps the load α_1 and communicates the remaining load (1-α_0-α_1) to its successor P_2. This process continues until the last processor obtains its share of load α_N. Each of the $N+1$ processors in the network are equipped with front ends. That is, each processor can compute its own fraction of load and communicates the rest of the load to its successor simultaneously. Consider the case where each processor begins to compute its fraction of load at the moment that it finishes receiving its own data. An important observation in considering linear networks is that, each processor in the network (except for the last processor) receives the fraction of load that is not only its own share of the load but also the fraction of loads that belongs to all of the rest of the processors that are beyond it.

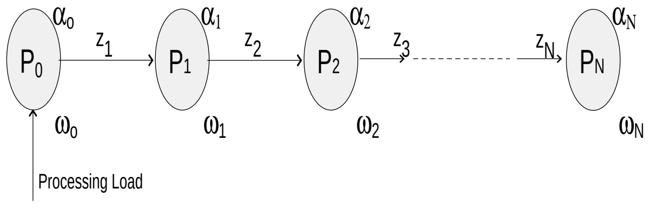


Fig. 1. Linear daisy chain network with front end processors.

This process of communication and load distribution is shown through a Gantt-chart-like timing diagram in Fig. 2. This protocol is referred to a "cut through switching" as the load fragment of a node's right neighbor is retransmitted by the node to its right neighbor once it is received. That is, the node does not wait for the entire load to be received before commencing retransmission.

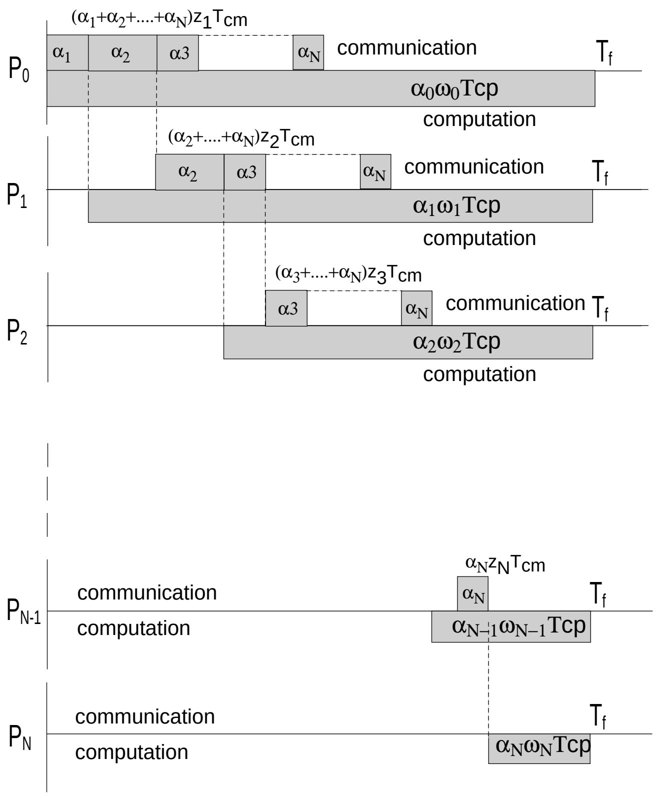

$(\alpha_1+\alpha_2+....+\alpha_N)z_1T_{cm}$
α_1 α_2 $\alpha3$ α_N communication
T_f
P_0
$\alpha_0\omega_0Tcp$
computation
$(\alpha_2+....+\alpha_N)z_2T_{cm}$
α_2 $\alpha3$ α_N communication
T_f
P_1
$\alpha_1\omega_1Tcp$
computation
$(\alpha_3+....+\alpha_N)z_3T_{cm}$
$\alpha3$ α_N communication
T_f
P_2
$\alpha_2\omega_2Tcp$
computation
$\alpha_Nz_NT_{cm}$
P_{N-1}
communication
computation
α_N
T_f
$\alpha_{N-1}\omega_{N-1}Tcp$
P_N
communication
computation
T_f
$\alpha_N\omega_NTcp$

Fig. 2. Timing diagram: linear daisy chain network, with cut through switching and front end processors and heterogeneous links.

As shown from the timing diagram, communication time appears above the axis and computation time appears below the axis. It is also shown that the processors have the same finishing time T_f. This corresponds to the fact that, for a minimum time solution all processors must stop computing at the same time. Indeed, otherwise some work could be transferred from a busy processor to an idle one in order to improve the solution time. Based on this result one can write the following set of equations:

$$\alpha_0\omega_0 T_{cp} = \alpha_1 z_1 T_{cm} + \alpha_1\omega_1 T_{cp} \tag{1}$$
$$\alpha_1\omega_1 T_{cp} = \alpha_2(z_1 + z_2)T_{cm} + \alpha_2\omega_2 T_{cp} \tag{2}$$
$$\alpha_2\omega_2 T_{cp} = \alpha_3(z_2 + z_3)T_{cm} + \alpha_3\omega_3 T_{cp} \tag{3}$$

$$\alpha_{N-2}\omega_{N-2} T_{cp} = \alpha_{N-1}(z_{N-2} + z_{N-1})T_{cm} + \alpha_{N-1}\omega_{N-1} T_{cp} \tag{4}$$
$$\alpha_{N-1}\omega_{N-1} T_{cp} = \alpha_N(z_{N-1} + z_N)T_{cm} + \alpha_N\omega_N T_{cp} \tag{5}$$

In this paper, the objective in presenting the above set of equations is to find a continuous time Markov chain model with α_i's being analogous to the steady state probabilities and the communication and computation time parameters being accounted for as transition rates that satisfy a set of local balance equations which corresponds to the above set of equations. In this case the Markov chain model which satisfies the requirements given above is shown in Fig. 3.

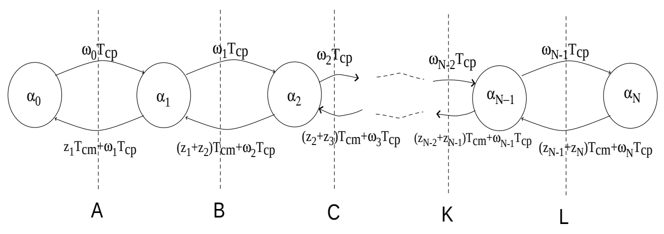


Fig. 3. Markov chain model for linear network with cut through switching and front end processors and heterogeneous links.

As shown in the Markov chain, using the local boundaries, one can write the following set of local balance equations as in the following manner. Using a balance equation at boundary A one can write:

$$\alpha_0\omega_0 T_{cp} = \alpha_1 z_1 T_{cm} + \alpha_1\omega_1 T_{cp} \tag{6}$$

At boundary B also one can write:

$$\alpha_1\omega_1 T_{cp} = \alpha_2(z_1 + z_2)T_{cm} + \alpha_2\omega_2 T_{cp} \tag{7}$$

Similarly, for boundaries K and L one can have the following equations, respectively:

$$\alpha_{N-2}\omega_{N-2} T_{cp} = \alpha_{N-1}(z_{N-2} + z_{N-1})T_{cm} + \alpha_{N-1}\omega_{N-1} T_{cp} \tag{8}$$
$$\alpha_{N-1}\omega_{N-1} T_{cp} = \alpha_N(z_{N-1} + z_N)T_{cm} + \alpha_N\omega_N T_{cp} \tag{9}$$

This set of equations directly correspond to the set of equations that are derived from the Gantt-chart-like timing diagram. This new modeling tool combines both the equations and diagram into a Markov chain which is simple and compact. Note that the pattern of transition rate labeling is unusual for a Markov chain model in telecommunications and networking applications.

The load scheduling strategy discussed above is one of the many that may be modeled using divisible load theory. Now consider the same network but with a different load scheduling strategy, store and forward switching and homogeneous links. In this case it will be assumed that each processor in the network starts its computation after receiving all the fractions of load that belong to all the processors that are beyond it. This process of load distribution is shown in Fig. 4. In the previous case the processor starts its computation immediately after receiving its own share. Again, it is assumed that processors have front end processors. The set of equations for solving for the minimum finish time can be written as:

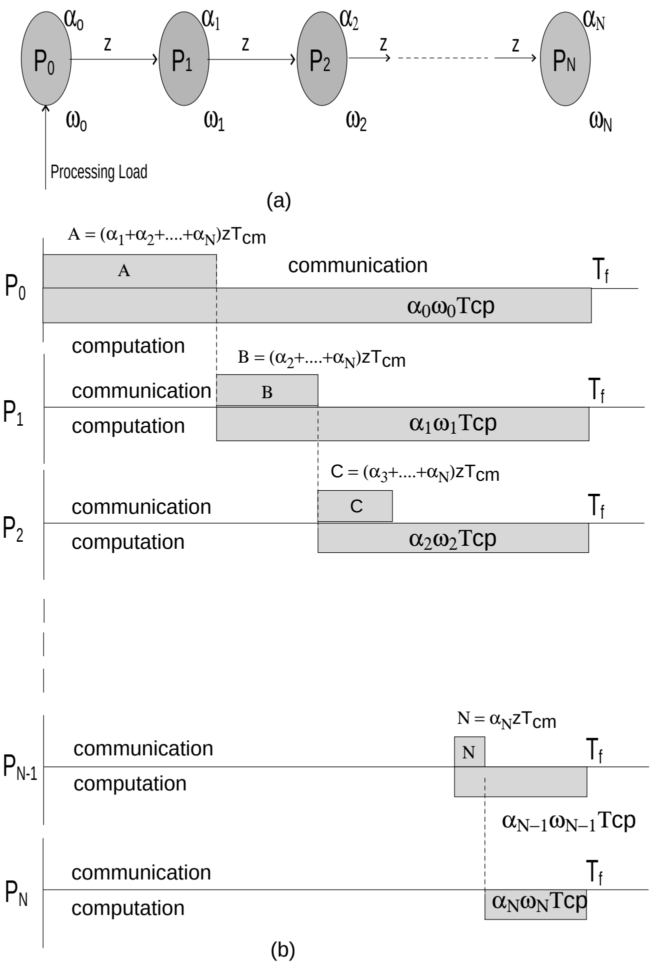


Fig. 4. Linear daisy chain network (a) with store and forward switching, front end and homogeneous links (b) timing diagram.

$$\alpha_0\omega_0T_{cp} = (\alpha_1 + \alpha_2 + ... + \alpha_N)zT_{cm} + \alpha_1\omega_1T_{cp} \tag{10}$$
$$\alpha_1\omega_1T_{cp} = (\alpha_2 + \alpha_3 + ... + \alpha_N)zT_{cm} + \alpha_2\omega_2T_{cp} \tag{11}$$

$$\alpha_i\omega_iT_{cp} = (\alpha_{i+1} + \alpha_{i+2} + ... + \alpha_N)zT_{cm} + \alpha_{i+1}\omega_{i+1}T_{cp} \tag{12}$$

$$\alpha_{N-1}\omega_{N-1}T_{cp} = \alpha_N zT_{cm} + \alpha_N\omega_NT_{cp} \tag{13}$$

The corresponding Markov chain model which has the same set of local balance equations as the above set of equations is shown in Fig. 5. In this case the communication speeds need to be homogeneous in order for this chain to be equivalent. This provides a counterexample to show that not every divisible

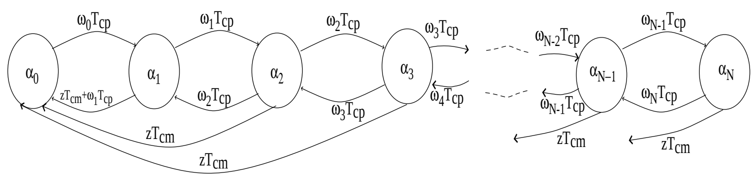


Fig. 5. Markov chain model for linear daisy chain with store and forward switching, front end and homogeneous links.

load model has a corresponding Markov chain.

4 Single level tree networks

4.1 Single level tree without front end processors

A single level tree network of $N+1$ processors and N links is shown in Fig. 6. The root processor P_0, where the load originates keeps its fraction of the total load for itself to compute and distributes sequentially the remaining load to its child processors at the lower level. Each processor in the network is assumed to have no front end processor. That is, the root processor will first finish communicating all of the load to be transmitted to the lower level before it starts computing its own fraction of load. The terminal processors start computing only after completely receiving their respective fraction of loads (known as staggered start). Note that if all z_i's have the same numerical value, one has a bus network model. The timing diagram showing the process of load distribution for a single level tree network with out front end processors is shown Fig. 6.

Now one can write the following set of equations for solving for the optimal solution time as:

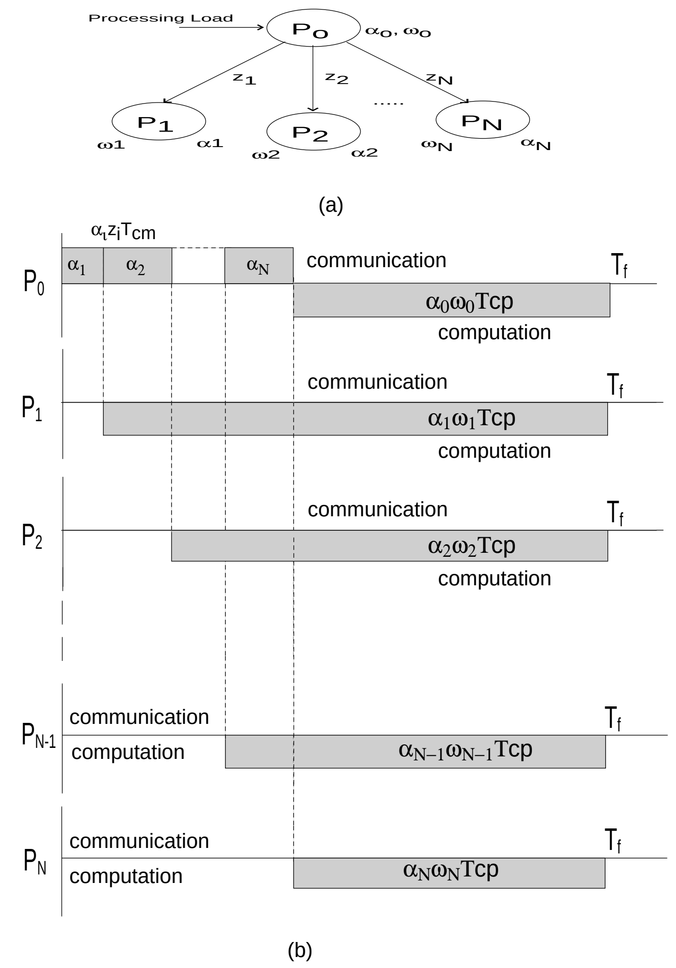


Fig. 6. Single level tree network - (a) network topology (b)Timing diagram without front end at nodes.

$$\alpha_0\omega_0 T_{cp} = \alpha_N \omega_N T_{cp} \tag{14}$$

$$\alpha_1\omega_1 T_{cp} = \alpha_2 z_2 T_{cm} + \alpha_2\omega_2 T_{cp} \tag{15}$$

$$\alpha_i\omega_i T_{cp} = \alpha_{i+1} z_{i+1} T_{cm} + \alpha_{i+1}\omega_{i+1} T_{cp} \tag{16}$$

$$\alpha_{N-2}\omega_{N-2} T_{cp} = \alpha_{N-1} z_{N-1} T_{cm} + \alpha_{N-1}\omega_{N-1} T_{cp} \tag{17}$$

$$\alpha_{N-1}\omega_{N-1} T_{cp} = \alpha_N z_N T_{cm} + \alpha_N\omega_N T_{cp} \tag{18}$$

The corresponding Markov chain model which has the same set of local balance equations as the above set of equations is shown in Fig. 7. Note that state 0 is a neighbor of state N, which is unusual for a typical telecommunications Markov chain model.

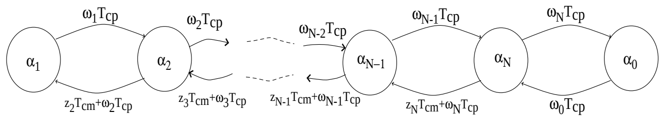


Fig. 7. Markov chain model for single level tree network without front end processors.

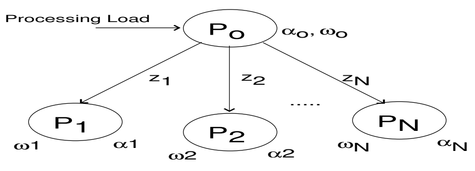
4.2 Single level tree networks with front end processors

Another single level tree network of $N+1$ processors and N links is shown in Fig. 8. As discussed earlier, the root processor P_0, where the load originates keeps its fraction of the total load for itself to compute and distributes the remaining load to its child processors at the next lower level sequentially. The root processor in the network is equipped with a front end. That is, the root can compute its own fraction of load and communicates the rest of the load to each of its children simultaneously. In this case each processor begins to compute its fraction of load at the moment that it finishes receiving its data. The timing diagram of the process of load distribution for a single level tree network with front end processors is shown Fig. 8.

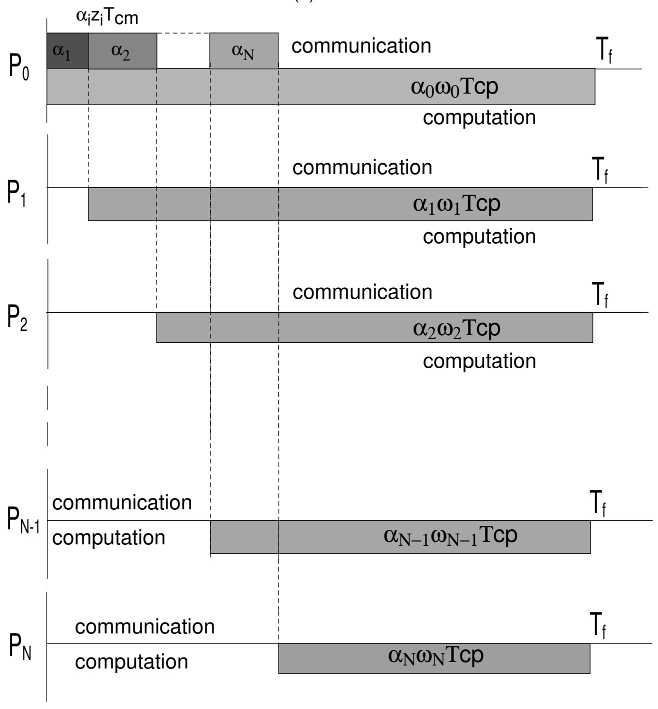
The set of equations for solving for the minimum finish time can be written as:

$$\alpha_0 \omega_0 T_{cp} = \alpha_1 z_1 T_{cm} + \alpha_1 \omega_1 T_{cp} \tag{19}$$
$$\alpha_1 \omega_1 T_{cp} = \alpha_2 z_2 T_{cm} + \alpha_2 \omega_2 T_{cp} \tag{20}$$

$$\alpha_i \omega_i T_{cp} = \alpha_{i+1} z_{i+1} T_{cm} + \alpha_{i+1} \omega_{i+1} T_{cp} \tag{21}$$



(a)



(b)

Fig. 8. Single level tree network - (a) network topology (b)Timing diagram for front end case.

$$\alpha_{N-2}\omega_{N-2}T_{cp} = \alpha_{N-1}z_{N-1}T_{cm} + \alpha_{N-1}\omega_{N-1}T_{cp} \tag{22}$$
$$\alpha_{N-1}\omega_{N-1}T_{cp} = \alpha_N z_N T_{cm} + \alpha_N \omega_N T_{cp} \tag{23}$$

The corresponding Markov chain model which has the same set of local balance equations as the above set of equations is shown in Fig. 9.

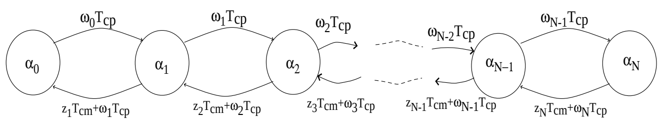

α_0
α_1
α_2
α_{N-1}
α_N
$\omega_0 T_{cp}$
$\omega_1 T_{cp}$
$\omega_2 T_{cp}$
$\omega_{N-2} T_{cp}$
$\omega_{N-1} T_{cp}$
$z_1 T_{cm}+\omega_1 T_{cp}$
$z_2 T_{cm}+\omega_2 T_{cp}$
$z_3 T_{cm}+\omega_3 T_{cp}$
$z_{N-1} T_{cm}+\omega_{N-1} T_{cp}$
$z_N T_{cm}+\omega_N T_{cp}$

Fig. 9. Markov chain model for single level tree network with front end processor at root.

5 Two level tree networks

Consider a two level binary tree network of communicating processors as depicted in Fig. 10. As can be seen from the figure, there are three types of nodes: root, intermediate and terminal nodes. The root is the node where the processing load originates. Then there are intermediate nodes which can be viewed as parents of the lower level nodes with which they have direct communication. The terminal nodes are nodes that have no child processors and hence can only be children nodes. In this section, consider the case where the communication between the root processor and the intermediate nodes is concurrent and the communication between the intermediate nodes and the terminal nodes is sequential. As in section 3, "cut through switching" is used at the intermediate nodes. These assumptions are made to show the ubiquity of the divisible load schedule and Markov chain equivalence. This process of load distribution and communication is shown in Fig. 11.

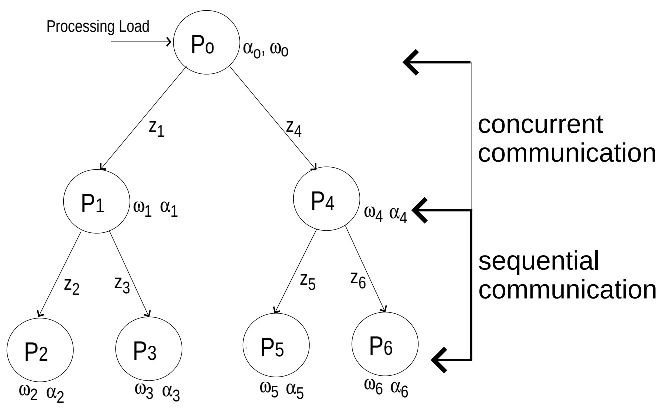


Fig. 10. Two level tree network with front end processors.

The set of equations based on the minimum finish time can be written as:
For branch 1 (linking P_0 to P_1), one can write:

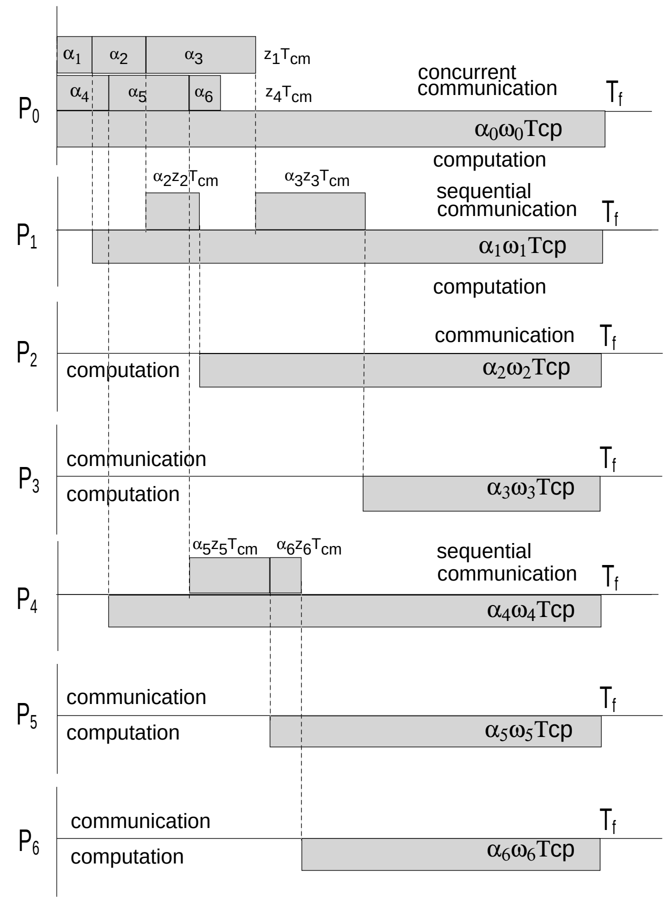


Fig. 11. Timing diagram of a two level tree network with front end processors.

$$\alpha_0\omega_0T_{cp} = \alpha_1z_1T_{cm} + \alpha_1\omega_1T_{cp} \tag{24}$$
$$\alpha_1\omega_1T_{cp} = \alpha_2(z_1 + z_2)T_{cm} + \alpha_2\omega_2T_{cp} \tag{25}$$
$$\alpha_2z_2T_{cm} + \alpha_2\omega_2T_{cp} = \alpha_3(z_1 + z_3)T_{cm} + \alpha_3\omega_3T_{cp} \tag{26}$$

Similarly, for branch 4 (linking P_0 to P_4):

$$\alpha_0\omega_0T_{cp} = \alpha_4z_4T_{cm} + \alpha_4\omega_4T_{cp} \tag{27}$$
$$\alpha_4\omega_4T_{cp} = \alpha_5(z_4 + z_5)T_{cm} + \alpha_5\omega_5T_{cp} \tag{28}$$
$$\alpha_5\omega_5T_{cp} = \alpha_6z_6T_{cm} + \alpha_6\omega_6T_{cp} \tag{29}$$

Note that the two branches 1 and 4 have, as is possible, two structurally different local balance equations based on the sequence of their fraction of loads. In the first case the fraction of loads are arranged in such a way that load fractions to children are in increasing order of load fragment size. Whereas in the second branch the load fractions to children are arranged in decreasing order of load fragment size.

The corresponding Markov chain model which has the same set of local balance equations as the above set of equations is shown in Fig. 12. Note that state 0 is at the center of the Markov chain and states α_1, α_2, and α_3 which correspond to branch 1 are the left side neighbors of state 0. On the other hand states α_4, α_5, and α_6 which correspond to branch 4 are shown to be the right side neighbors of state 0. This is, again, unusual for the typical telecommunications Markov chain model.

$$\begin{tikzcd}
\alpha_3 \arrow[r, bend left, "(z_1+z_3)Tcm+\omega_3 Tcp"] & \alpha_2 \arrow[l, bend left, "z_2 Tcm+\omega_2 Tcp"] \arrow[r, bend left, "(z_1+z_2)Tcm+\omega_2 Tcp"] & \alpha_1 \arrow[l, bend left, "\omega_1 Tcp"] \arrow[r, bend left, "z_1 Tcm+\omega_1 Tcp"] & \alpha_0 \arrow[l, bend left, "\omega_0 Tcp"] \arrow[r, bend left, "\omega_0 Tcp"] & \alpha_4 \arrow[l, bend left, "z_4 Tcm+\omega_4 Tcp"] \arrow[r, bend left, "\omega_4 Tcp"] & \alpha_5 \arrow[l, bend left, "(z_4+z_5)Tcm+\omega_5 Tcp"] \arrow[r, bend left, "\omega_5 Tcp"] & \alpha_6 \arrow[l, bend left, "z_6 Tcm+\omega_6 Tcp"]
\end{tikzcd}$$

Fig. 12. Markov chain model for two level tree network with front end processors.

As shown from the model considered earlier each processor can have at most two child processors. However, tree networks can have more than two child processors per node. Fig. 13 depicts the case where the root processor has N child processors, $P_1, P_2, ..., P_N$. Each of the N processors is also a parent processor of the lower level processors or terminal nodes with which it has direct communication. It is assumed that each of these parents has M child processors. As shown in the figure the notation used for the first level of the network has only one digit showing the branch number. On the other hand, for the second level there are two digits used for notation. The first digit shows the branch number of the processor and the second digit shows child number within the same branch. That is, the notation $P_{1,1}$ indicates that the parent processor is from branch 1 and the second digit shows this is the first child.

Similarly, $P_{N,M}$ denotes the M^{th} child of the N^{th} parent processor.

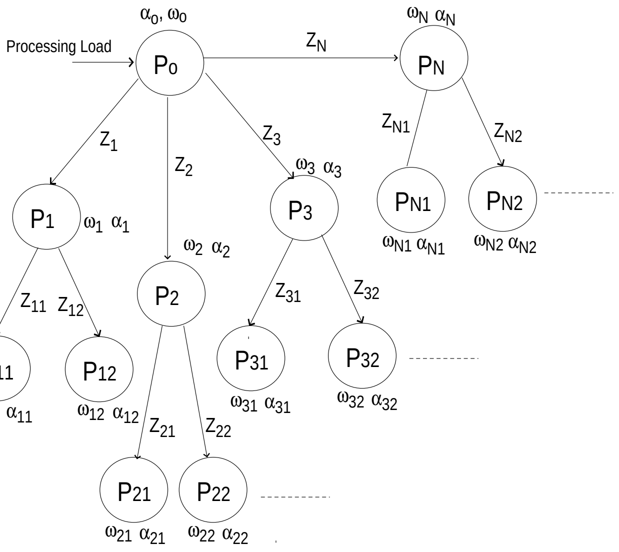


Fig. 13. Two level tree network with front end processors and $N > 2$ children.

The timing diagram showing the process of load distribution in this type of network topology is shown in Fig. 14. In this case we again consider the case where the communication between the root processor and the intermediate nodes is concurrent, whereas the communication between the intermediate nodes and the terminal nodes is sequential. The set of equations for obtaining the minimum finish time can be written as:

For branch 1:

$$\alpha_0\omega_0T_{cp} = \alpha_1 z_1 T_{cm} + \alpha_1\omega_1T_{cp} \tag{30}$$

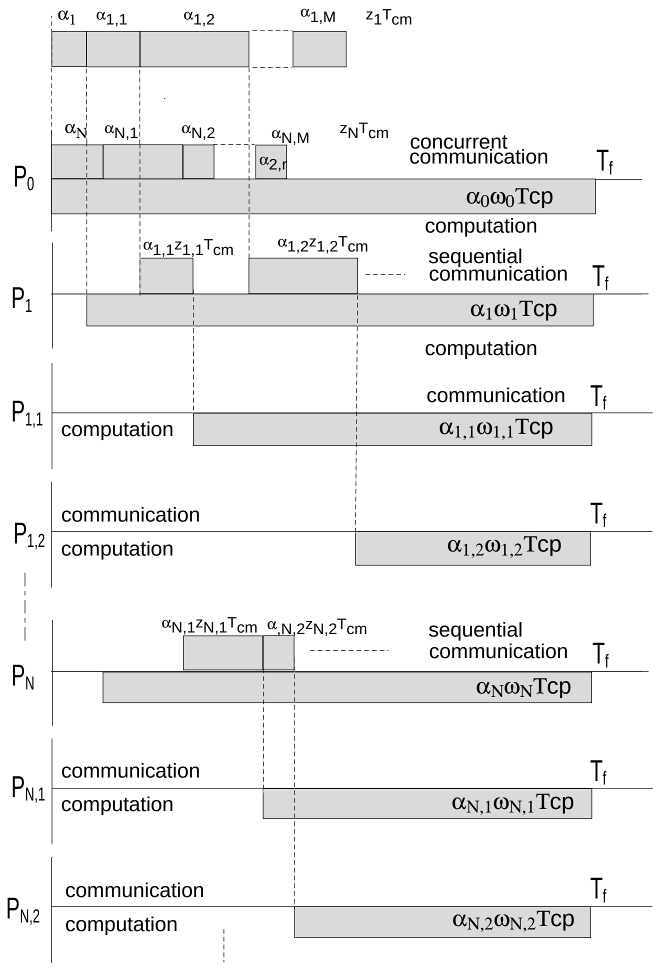
$$\alpha_1\omega_1T_{cp} = \alpha_{1,1}(z_1 + z_{1,1})T_{cm} + \alpha_{1,1}\omega_{1,1}T_{cp} \tag{31}$$

$$\alpha_{1,1}\omega_{1,1}T_{cp} + \alpha_{1,1}z_{1,1}T_{cm} = \alpha_{1,2}(z_1 + z_{1,2})T_{cm} + \alpha_{1,2}\omega_{1,2}T_{cp} \tag{32}$$

For any child i where $i > 2$ in the lower level of the first branch, and with increasing load fragment size, one can write:

$$\alpha_{1,i-1}\omega_{1,i-1}T_{cp} + \alpha_{1,i-1}z_{1,i-1}T_{cm} = \alpha_{1,i}(z_1 + z_{1,i})T_{cm} + \alpha_{1,i}\omega_{1,i}T_{cp} \tag{33}$$

Similarly, for branch N, one can write the following:



α_1 $\alpha_{1,1}$ $\alpha_{1,2}$ $\alpha_{1,M}$ z_1T_{cm}
α_N $\alpha_{N,1}$ $\alpha_{N,2}$ $\alpha_{N,M}$ z_NT_{cm}
concurrent communication
$\alpha_{2,r}$
T_f
P_0
$\alpha_0\omega_0Tcp$
computation
$\alpha_{1,1}z_{1,1}T_{cm}$ $\alpha_{1,2}z_{1,2}T_{cm}$
sequential communication
P_1
$\alpha_1\omega_1Tcp$
computation
$P_{1,1}$
communication
computation
$\alpha_{1,1}\omega_{1,1}Tcp$
$P_{1,2}$
communication
computation
$\alpha_{1,2}\omega_{1,2}Tcp$
$\alpha_{N,1}z_{N,1}T_{cm}$ $\alpha_{N,2}z_{N,2}T_{cm}$
sequential communication
P_N
$\alpha_N\omega_NTcp$
$P_{N,1}$
communication
computation
$\alpha_{N,1}\omega_{N,1}Tcp$
$P_{N,2}$
communication
computation
$\alpha_{N,2}\omega_{N,2}Tcp$

Fig. 14. Timing diagram of a tree network with front end processors and $N > 2$ children.

$$\alpha_0\omega_0T_{cp} = \alpha_N z_N T_{cm} + \alpha_N\omega_N T_{cp} \tag{34}$$
$$\alpha_N\omega_N T_{cp} = \alpha_{N,1}(z_N + z_{N,1})T_{cm} + \alpha_{N,1}\omega_{N,1}T_{cp} \tag{35}$$
$$\alpha_{N,1}\omega_{N,1}T_{cp} = \alpha_{N,2}z_{N,2}T_{cm} + \alpha_{N,2}\omega_{N,2}T_{cp} \tag{36}$$

Again eqns. (42) and (46) are structurally different because of the different order of load fragment sizes in this example realization.

For any child i, where $i > 2$ in the lower level of the N^{th} branch, and with decreasing load fragment size, one can write:

$$\alpha_{N,i-1}\omega_{N,i-1}T_{cp} = \alpha_{N,i}z_{N,i}T_{cm} + \alpha_{N,i}\omega_{N,i}T_{cp} \tag{37}$$

The corresponding Markov chain model which has the same set of local balance equations as the above set of equations is shown in Fig. 15.

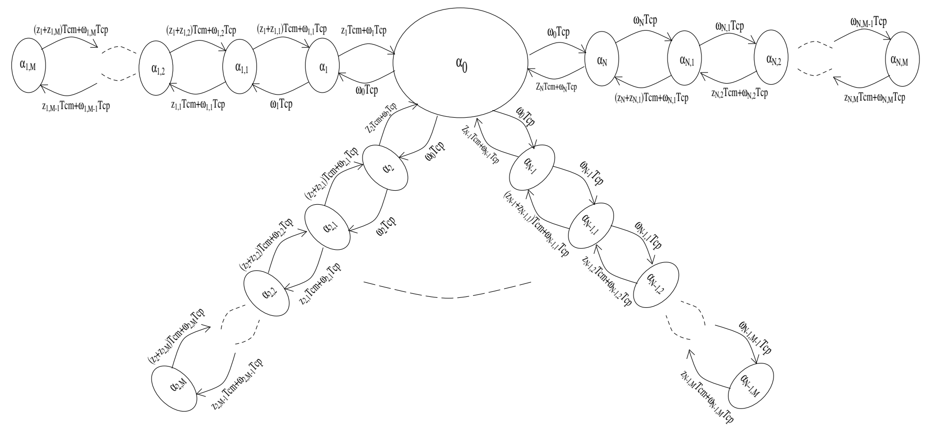


Fig. 15. Markov chain model for two level tree network with front end processors and $N > 2$ children.

6 Open Problems

The unification introduced between divisible load theory and continuous time Markov chain models, we believe is a potential cross-over of technology and modeling from one discipline to another.

We mention two open problems, one a specific one and one a general one, in that order. It is well known that for any arbitrary Markov chain, the steady state probabilities can be computed by solving the associated set of linear global balance equations. The set of global balance equations equate the net inflow of probability flux at a state to the net out flow from the same state. But the numerical solution techniques for N state arbitrary Markov chains can involve a computational complexity of O(N^3), allowing only Markov chains of modest size to be exactly solved. However since the original work of Jackson in 1957 [21], and later Gordon and Newel in 1967 [15], researchers are able to produce elegant and tractable analytic solutions of the product form type from different complex system models. That is, for this product form class of queueing networks any state equilibrium probability is a product of system parameters and a reference probability (local balance equations). It is an open question as to what type of divisible load scheduling model, if any, corresponds to a product form solution of order greater than one.

A general open problem of interest is whether a combined stochastic (queueing) and deterministic (scheduling) model is possible with a single solution. This could be of use in situations such as multiple divisible jobs queueing in buffers to be scheduled for transmission and processing on multiple links and processors. It is not clear at this point whether joint stochastic/deterministic modeling is possible. However, it is an exciting open research problem.

7 Conclusion

In this paper an alternative model for performance evaluation of divisible load schedules using Markov chain models is presented. The model is based on the principle of local balance equations. We have provided Markov chain models for linear daisy chains, single level trees (with and without front end processors) and two level tree networks. It was found that these models are relatively simple and compact. Further areas of research would include multi level tree networks. Moreover it would be interesting to explore how to extend this result to networks that are more complex including hypercubes and two dimensional meshes.

The many commonalities between queueing theory and divisible load theory (and even electric circuit theory) show that linear models share a common foundation but differ in their detailed features. This latter point is emphasized by the fact that not every parameterized divisible load scheduling model has a corresponding Markov chain. This richness of the linear modeling paradigm, as well as its tractability and breadth of applications will make divisible load scheduling an intriguing field of study well into the future.

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