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Theory of the photon-drag effect in a two-dimensional electron gas

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Intersubband transitions stimulated by an electromagnetic wave which has a momentum in the plane of a two-dimensional electron gas (2DEG) are accompanied by a "photon-drag" current due to the momentum imparted to electrons by the absorbed photons. In a high-mobility 2DEG at low temperatures, an enhanced effect occurs owing to the difference in the momentum relaxation times in the ground and the excited subbands. A theory of this effect is developed with particular attention paid to the geometrical aspects of proposed experiments. The enhanced effect is considered in two limits, corresponding to two dominant mechanisms of line broadening in the intersubband absorption spectra (the collision and the Doppler linewidths). The photon-drag effect permits a new type of spectroscopy containing information about the momentum-relaxation kinetics in 2D subbands. Moreover, it allows the implementation of novel far-infrared detectors. Fundamental limits on the quantum efficiency of such detectors are discussed.

I. INTRODUCTION

The photon-drag effect is entirely owing to the existence of a photon momentum. Classically, the momentum carried by electromagnetic waves manifests itself through a radiation pressure. The latter can be easily determined if one knows the reflected and the transmitted fractions of incident radiation. Consequently, light-pressure experiments deliver only part of the information that can be (much more easily and accurately) extracted from the usual light-reflection and absorption experiments. On the other hand, if the absorption of radiation is due to free carriers or to optical transitions leading to the formation of free carriers, then these carriers acquire a directed motion caused by the absorbed photon momentum. The magnitude of this current carries additional information about the energy spectrum of electrons and various scattering mechanisms.

The low-frequency ($\omega\tau \ll 1$, where τ is the electron-momentum relaxation time, and ω the angular frequency of the electromagnetic wave) photon-drag effect (PDE) was first considered by Barlow,[1] who described it as an ac Hall effect due to the electric and magnetic field vectors of the wave. If we neglect the difference between the usual electron mobility μ and the Hall mobility, then Barlow's result can be derived on the basis of a very simple argument, cf. Fig. 1. Consider a flux Φ of monochromatic photons of energy $\hbar\omega$ and momentum $\hbar q$. For simplicity, we assume an isotropic medium of permittivity ϵ, so that $\omega/q = c/\sqrt{\epsilon}$, and $\mathbf{S} = (\Phi/\hbar\omega)\mathbf{q}/q$, where $\mathbf{S}$ is the radiation energy flux (the Poynting vector) in the medium. Let α be the free-electron absorption coefficient. Then $\alpha\hbar q\Phi$ represents the rate of momentum transfer to the electronic system per unit volume, i.e., the drag-force density. If the volume density of free electrons is N, then one can say that these electrons are acted upon by an effective electric field $\mathbf{E}^* = -\alpha\hbar\mathbf{q}\Phi/eN$. This field gives rise to a current density

$$\mathbf{J} = eN\mu\mathbf{E}^* = -\alpha\Phi\mu\hbar\mathbf{q} = -\frac{\alpha\mu\mathbf{S}}{c/\sqrt{\epsilon}} . \quad (1.1)$$

Although the simplified approach, used in the derivation of (1.1), does not take into account the fact that the process of absorption requires the participation of phonons and/or impurities (as is well known, a truly free-electron gas does not absorb photons), nevertheless for $\omega\tau \ll 1$ [where τ is related to electron mobility by the usual $\mu = (e/m)\tau$], it leads to a result practically identical to that obtained by more rigorous methods.[2,3] However, this approach loses validity in the quantum frequency range, when $\hbar\omega \gtrsim kT$ (it is already invalid in the "classical" regime $\hbar\omega < kT$ if $\omega\tau > 1$);[2,3] Eq. (1.1) is totally inadequate for the consideration of optical transitions between quantum subbands. The PDE based on such transitions was independently discovered by two groups.[4,5] In these early experiments, the optical transitions were realized between the valence subbands in p-type germanium. Subsequently, the PDE was studied in other semiconductors for a variety of optical transitions, including

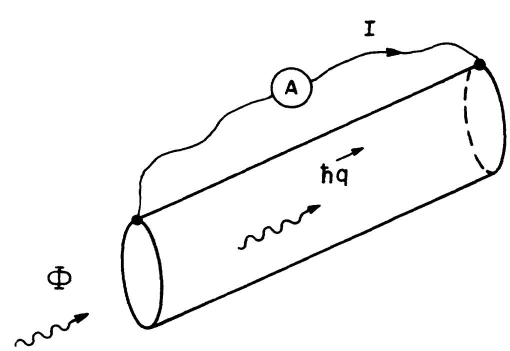

FIG. 1. Schematic illustration of a photon-drag effect experiment.

intraband transitions,[6] one- and two-photon interband transitions,[7,8] and impurity-ionization transitions.[9,10]

In the case of optical transitions between two quantum subbands, the PDE current is formed by a combination of the current due to electrons in the excited subband and the current carried by the ground-subband electronic system, driven from the equilibrium by a velocity-selective excitation. These two currents may have the same or opposite sign, and the net PDE current can be directed either along or opposite to the incident photon flux. (One can also have a transverse effect when the current and flux are not collinear. Such an effect can take place even in crystals possessing a cubic symmetry provided the electron energy spectrum is not spherically symmetric.[11-13]) Within each subband, the net currents are also determined by differences of oppositely directed currents—those along and opposite to the direction of light. In special cases, usually realized in a narrow range of the incident radiation frequency, one of the above currents may become dominant; the net PDE current is then radically different from the current outside the "resonant" range.

One example of such a resonance was predicted by Grinberg and Udod[14] for the case of PDE in bulk *p*-type Ge, when optical transitions occur between the valence subbands. Such transitions are schematically illustrated in Fig. 2(a). Because of the conservation laws of energy and quasimomentum, the energies E_1^+ and E_2^+ of nonequilibrium holes with $\mathbf{k}\cdot\mathbf{q}>0$ are larger than the corresponding energies E_1^- and E_2^- of holes with $\mathbf{k}\cdot\mathbf{q}<0$. Consider the PDE current excited in the light-hole subband. The resonance occurs when

$$E_2^- < \hbar\omega_{\rm opt} < E_2^+ \ , \tag{1.2}$$

where $\hbar\omega_{\rm opt}$ is the optical phonon energy. At sufficiently low temperatures, $kT \ll \hbar\omega_{\rm opt}$, only the light holes with $\mathbf{k}\cdot\mathbf{q}>0$ will interact with optical phonons, leading to substantially shorter momentum-relaxation time for such holes. Consequently, the current in the light-hole subband will be dominated by holes with $\mathbf{k}\cdot\mathbf{q}<0$. This type of "resonant" PDE is brought about by the sharp dependence of the momentum-relaxation time on energy. A similar resonance can be realized with interband optical transitions.[7]

Another type of resonance in bulk *p*-type germanium occurs when the excitation of current carriers is selective with respect to their quasimomentum and the temperature is sufficiently low to ensure a sharp boundary at the Fermi energy E_F in the distribution of holes. As is evident from Fig. 2(a), if the excitation energy $\hbar\omega$ is such that the energies E_1^-, E_1^+ and E_F satisfy the inequality

$$E_1^- < E_F < E_1^+ \ , \tag{1.3}$$

then only the holes with $\mathbf{k}\cdot\mathbf{q}<0$ will make optical transitions. It has been shown, theoretically, that in the resonant range the PDE current can be enhanced by almost 3 orders of magnitude.[15]

Situations similar to the second type of resonance can arise in the case of optical transitions between equidistant bands, such as those in a two-dimensional electron gas in inversion layers or quantum wells, or Landau subbands in a magnetic field. In this case, the energy and momentum conservation laws in the linear in $\mathbf{q}$ approximation lead to the result that the excitation frequency must satisfy the Doppler relation

$$\omega=\omega_{21}\left[1+\frac{(\mathbf{v}\cdot\mathbf{q})\sqrt{\epsilon}}{cq}\right]\approx\omega_{21}\left[1+\frac{\mathbf{v}\cdot\mathbf{q}}{\omega_{21}}\right] , \tag{1.4}$$

where $\mathbf{v}$ is the electron velocity and $\hbar\omega_{21}$ the energy gap between the two parallel bands. Equation (1.4) implies that electrons with $\mathbf{v}\cdot\mathbf{q}>0$ can absorb only those photons whose energy $\hbar\omega>\hbar\omega_{21}$, cf. Fig. 2(b). Conversely, if $\hbar\omega<\hbar\omega_{21}$, then only electrons with $\mathbf{v}\cdot\mathbf{q}<0$ will be excited. If the momentum-relaxation times in the two subbands are different, then the velocity-selective excitation provided by the Doppler effect will give rise to a substantially enhanced PDE current. Such an effect was first suggested by Dykhne *et al.*,[16] who discussed it by analogy with the known effect of *light-induced drift* of neutral atoms in the gas phase.[17,18] Recently,[19] one of us considered the PDE in a degenerate two-dimensional elec-

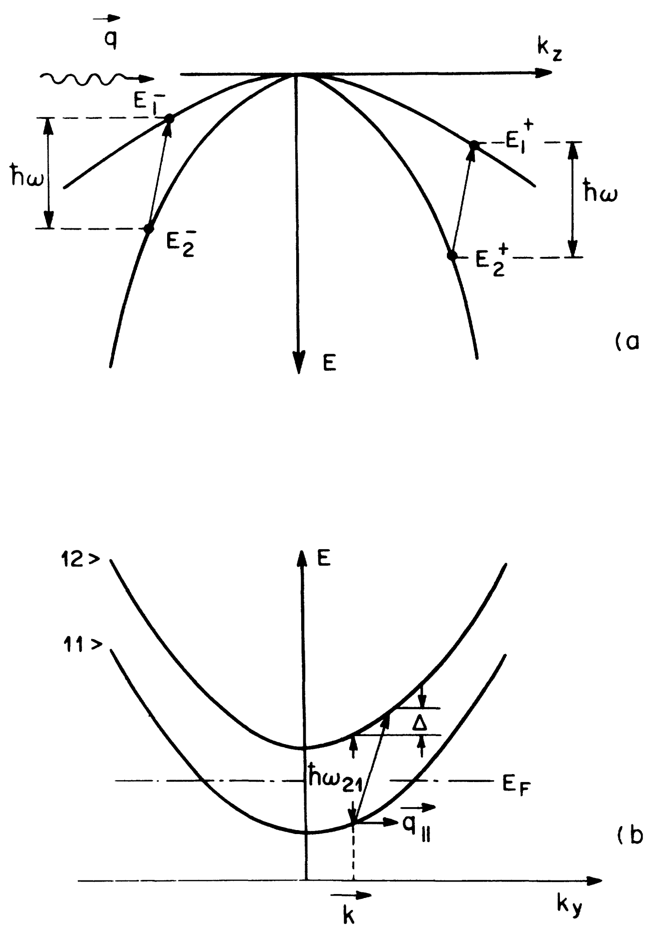

FIG. 2. Illustration of the optical transitions between quantum subbands. (a) Transitions in bulk *p*-type Ge between the heavy-hole and the light-hole valence subbands. (b) Transitions between equidistant subbands in a two-dimensional electron gas; $\Delta_q \equiv \hbar\mathbf{v}\cdot\mathbf{q}=\hbar^2\mathbf{k}\cdot\mathbf{q}_{\parallel}/m$.

tron gas (2DEG) in the presence of a collisional level broadening. It was shown that the drag-current spectrum contains new information about the momentum-relaxation kinetics in 2D subbands.

The purpose of this work is to further develop a microscopic theory of the PDE in quantum wells (QW's). On the basis of a solution of the kinetic equation, we shall generalize the results obtained earlier[19] to the case when the momentum-relaxation times *within each 2D subband* themselves depend on the electron kinetic energy; moreover, the present results are valid whether or not the electronic system is degenerate. At the same time we shall extend the consideration of the geometrical aspects of the problem. The earlier work assumed a specific experimental arrangement with the incident radiation propagating as a transverse magnetic wave of a waveguide formed by multiple quantum wells; here we shall discuss two additional illumination schemes and present expressions which should be convenient for a comparison with experiments. We shall also discuss the fundamental limitations on the sensitivity of PDE-based photodetectors.

II. GEOMETRY OF THE PROBLEM

Photon-drag current in a QW can be induced using three distinct illumination arrangements, schematically illustrated in Fig. 3. In the waveguide scheme,[19] [Fig. 3(a)], it is natural to characterize the absorption of light by an absorption coefficient $\alpha(\omega)$, whereas for oblique illumination schemes, [Figs. 3(b) and 3(c)], a more convenient characteristic is provided by an "absorbed fraction" coefficient $\beta(\omega)$, which is defined as the ratio of the absorbed and the incident radiation powers (cf. the Appendix). In what follows, we shall distinguish the quantities describing one or the other of the illumination schemes by affixing a subscript *a*, *b*, or *c*, corresponding to the schemes in Figs. 3(a), 3(b), and 3(c), respectively.

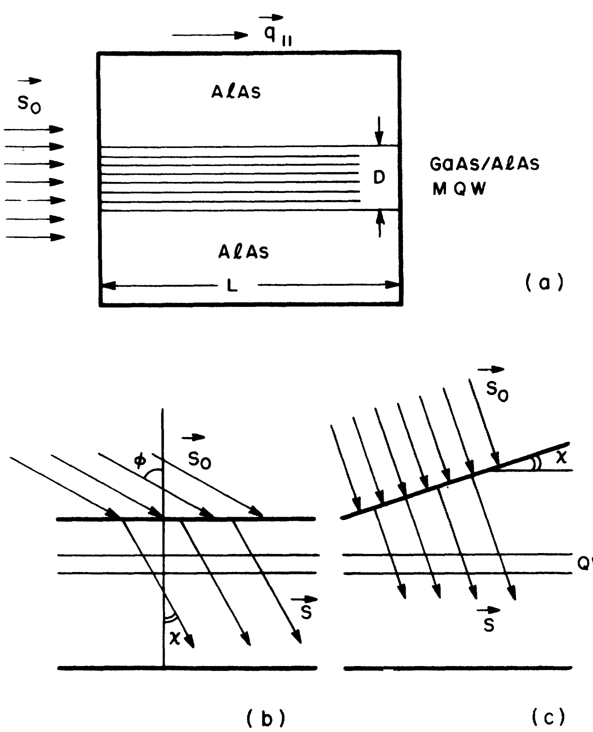

FIG. 3. Illustration of the three illumination schemes discussed in this work. (a) Waveguide scheme; infrared radiation travels as a TM wave of a dielectric waveguide formed by a multiple-QW core and two cladding layers of lower refractive index. (b), (c) Oblique illumination schemes, the radiation is polarized with the electric field vector lying in the plane of incidence; in the scheme (c) the incident light is normal to a crystal surface cut at an oblique angle to the QW.

Besides the incident angle, an important factor influencing the absorption process is the polarization of the infrared wave propagating in the crystal, since only the normal to the QW component E_z of the electric field can induce the intersubband transitions. Considering the oblique schemes of illumination, we shall be assuming that the incident radiation is polarized with the electric field lying within the plane of incidence. In the waveguide scheme, the appropriate mode of radiation is the transverse magnetic (TM) wave. An important geometric parameter affecting $\alpha(\omega)$ in this case is the degree of confinement of light.

$$\Gamma = 2\pi^2 (D/\lambda_0)^2 (\epsilon_{\rm core} - \epsilon_{\rm clad}) , \tag{2.1}$$

which is the fraction of the integrated intensity of the light wave propagating within the absorbing core of thickness D. To implement this scheme, it is virtually imperative to use a *multiple-QW* core.[19]

Next, consider an oblique illumination as in Fig. 3(b). Denote the wave vector of incident radiation (in vacuum) by $\mathbf{q}_0$ and that entering the QW by $\mathbf{q}$. (Here we shall neglect the small difference in the refractive index between the QW and the surrounding material.) Let $\mathbf{S}_0(\omega)$ be the spectral density of the incident electromagnetic flux in the direction $\mathbf{q}_0$ and $\mathbf{S}(\omega)$ the analogous quantity for the QW. The corresponding densities *per unit area* of the QW are given by

$$\begin{aligned} S_{0,z}(\omega) &= |\mathbf{S}_0(\omega)| \cos\phi , \\ S_z(\omega) &= |\mathbf{S}(\omega)| \cos\chi , \end{aligned} \tag{2.2}$$

where ϕ and χ are the angles of incidence and refraction, respectively, related by Snell's law: $\sin\phi = \sqrt{\epsilon}\sin\chi$. According to the Fresnel equations,[20] one has

$$S_z(\omega) = S_{0,z}(\omega) \frac{4\sqrt{\epsilon}\cos\phi\cos\chi}{(\sqrt{\epsilon}\cos\phi + \cos\chi)^2} . \tag{2.3}$$

Whence, using (2.2) one obtains

$$|\mathbf{S}(\omega)| = |\mathbf{S}_0(\omega)| \frac{4\sqrt{\epsilon}\cos^2\phi}{(\sqrt{\epsilon}\cos\phi + \cos\chi)^2} . \tag{2.4}$$

For the illumination scheme of Fig. 3(c), the radiation flux density $\mathbf{S}(\omega)$ propagating in the medium at the same angle with respect to the QW as in Fig. 3(b), is related to the incident flux $\mathbf{S}_0(\omega)$ (assumed directed *normally* to the crystal surface) as follows:

$$|\mathbf{S}(\omega)| = |\mathbf{S}_0(\omega)| \frac{4\sqrt{\epsilon}}{(\sqrt{\epsilon}+1)^2} . \tag{2.5}$$

III. INTERSUBBAND ABSORPTION LINEWIDTH

Consider a QW with 2DEG of surface concentration n. The subband energy spectrum is of the form

$$E_{i,\mathbf{k}}=E_i+E_{\mathbf{k}} \ , \tag{3.1}$$

where i is the subband index, $\mathbf{k}$ is the 2D electron wave vector in the QW plane, $E_{\mathbf{k}}=\hbar^2k^2/2m$, and m is the electron effective mass. We shall confine ourselves to the situation when only the first subband ($i=1$) is occupied, i.e., $E_2-E_1-\pi\hbar^2 n/m \gg kT$. We are interested in optical transitions to the next subband ($i=2$). Such transitions were recently studied experimentally in GaAs quantum wells.[21-23] The observed sharp absorption peaks (7 meV full width at half maximum), indicate that the line broadening in those samples was governed by electron scattering processes. Such broadening can be represented in the form

$$\gamma(k,k')=\frac{1}{2\tau_1(k)}+\frac{1}{2\tau_2(k')} \ , \tag{3.2}$$

where $\tau_i(k)\equiv\tau_i(E_{\mathbf{k}})$ are the momentum-relaxation times in the subbands $i=1$ and 2, and the momenta k and k' are related by the conservation laws (cf. Fig. 1)

$$\begin{aligned} \mathbf{k}'&=\mathbf{k}+\mathbf{q}_{\parallel} \ , \\ E_{2,\mathbf{k}'}&=E_{1,\mathbf{k}}+\hbar\omega \ . \end{aligned} \tag{3.3}$$

In these equations, $\mathbf{q}_{\parallel}$ is the component of $\mathbf{q}$ in the plane of the QW.

It should be noted that even in a perfectly uniform system (no inhomogeneous broadening) one has, besides the collision-broadened linewidth (3.2), also a *"natural" width* $\Delta_{\mathbf{q}}$ of the intersubband absorption line—associated with the finite magnitude of the photon wave vector. Indeed, Eqs. (3.3) imply

$$\hbar\omega=\hbar\omega_{21}+\hbar\omega_{\mathbf{k}+\mathbf{q}_{\parallel},\mathbf{k}} \ ,$$

where (3.4)

$$\hbar\omega_{\mathbf{k}+\mathbf{q}_{\parallel},\mathbf{k}}=E_{2,\mathbf{k}+\mathbf{q}_{\parallel}}-E_{1,\mathbf{k}}\approx\frac{\hbar^2\mathbf{k}\cdot\mathbf{q}_{\parallel}}{m} \ .$$

Equations (3.4) imply (1.4), which shows that the natural linewidth is of purely Doppler nature and its magnitude is determined by the scatter in electron velocities. For a degenerate distribution the $\hbar\omega_{\mathbf{k}+\mathbf{q}_{\parallel},\mathbf{k}}$ varies in the range $\Delta_{\mathbf{q}}=\hbar^2k_Fq_{\parallel}/m$, which for $n=10^{12}$ cm^{-2}, $\hbar\omega\approx0.15$ eV, and $m\approx0.07m_0$ gives $\Delta_{\mathbf{q}}\approx1$ meV. For a scattering time of order $\tau\sim10^{-13}$ s, the lifetime broadening (3.2) is an order of magnitude higher: $\hbar\gamma\sim7$ meV. We shall, therefore, consider the limit $\hbar\gamma\gg\Delta_{\mathbf{q}}$ first.

IV. THE PHOTON-DRAG CURRENT IN THE LIMIT OF SMALL DOPPLER SHIFT $\Delta_q \ll \hbar\gamma$

In order to evaluate the photon-drag current, we must determine the variation of electron distribution functions $f_i(\mathbf{k})$ in the two subbands, $i=1, 2$, connected by the optical transitions. Photon-induced transitions drive the electronic system away from the equilibrium, while relaxation processes—which include both intrasubband and intersubband scattering—attempt to restore the equilibrium. We shall label the photon-induced variations of the electron distribution by a subscript "phot." These variations for the two subbands involved are related by

$$\left[\frac{\partial f_1(\mathbf{k})}{\partial t}\right]_{\text{phot}}=-\left[\frac{\partial f_2(\mathbf{k}+\mathbf{q}_{\parallel})}{\partial t}\right]_{\text{phot}} \ . \tag{4.1}$$

Let us denote the nonequilibrium part of the distribution functions by a tilde, $\tilde{f}_i(\mathbf{k})$. In this case, when $\omega\tau\gg1$, the kinetic equations for $\tilde{f}_1$ and $\tilde{f}_2$ become decoupled.[24] Physically, this occurs because in the high-frequency limit the elementary processes of the electron scattering and the optical transition can be considered independently of each other. In the relaxation-time approximation we can write

$$\begin{aligned} \tilde{f}_1(\mathbf{k})&=\tau_1(E_{\mathbf{k}})\left[\frac{\partial f_1(\mathbf{k})}{\partial t}\right]_{\text{phot}} \ , \\ \tilde{f}_2(\mathbf{k}+\mathbf{q}_{\parallel})&=\tau_2(E_{\mathbf{k}+\mathbf{q}_{\parallel}})\left[\frac{\partial f_1(\mathbf{k})}{\partial t}\right]_{\text{phot}} \ , \end{aligned} \tag{4.2}$$

where the energy arguments of τ_i are referred to the edges of respective subbands.

The contributions I_i of both subbands to the drag current are given by

$$\begin{aligned} I_1&=-\frac{2e}{L}\sum_{\mathbf{k}}\tau_1(E_{\mathbf{k}})\left[\frac{\partial f_1(\mathbf{k})}{\partial t}\right]_{\text{phot}}\frac{\hbar(\mathbf{k}\cdot\mathbf{q}_{\parallel})}{mq_{\parallel}} \ , \\ I_2&=\frac{2e}{L}\sum_{\mathbf{k}}\tau_2(E_{\mathbf{k}+\mathbf{q}_{\parallel}}) \\ &\quad\times\left[\frac{\partial f_1(\mathbf{k})}{\partial t}\right]_{\text{phot}}\frac{\hbar[(\mathbf{k}+\mathbf{q}_{\parallel})\cdot\mathbf{q}_{\parallel}]}{mq_{\parallel}} \ , \end{aligned} \tag{4.3}$$

where L is the length of the QW in the direction of $\mathbf{q}_{\parallel}$.

In the evaluation of $[\partial f/\partial t]_{\text{phot}}$, the Hamiltonian operator corresponding to the electron-photon interaction should be taken in the form $\hat{\mathbf{H}}_{\text{phot}}=(e/mc)(\hat{\mathbf{A}}\cdot\hat{\mathbf{p}})$, where $\hat{\mathbf{A}}$ is the vector potential for photons, because the optical transitions are to be described *including* effects due to the photon momentum.[25] A straightforward calculation leads to the following expressions:

$$\left[\frac{\partial f_1(\mathbf{k})}{\partial t}\right]_{\text{phot}}=-\frac{2\pi^2e^2f_{12}\omega_{21}f_1^0(E_{\mathbf{k}})}{\hbar cm\omega^2\sqrt{\epsilon}}S(\omega)\sin^2(\chi) \ , \tag{4.4}$$

where $\mathbf{k}$ is determined by the condition (3.4), $f_1^0(E_{\mathbf{k}})$ is the equilibrium distribution function for the electron occupation of the first subband, and

$$f_{12}\equiv(2m\omega_{12}/\hbar)\,|\langle1|z|2\rangle|^2$$

is the oscillator strength of the optical transition.

If the spectral width of the light source is much narrower than the linewidth of the absorption spectrum, $\Delta\omega_{\text{rad}}\ll\gamma$, then we can replace $S(\omega)$ in (4.4) by

$$S(\omega)=S^{\rm tot}\frac{\gamma(k,\,|\mathbf{k}+\mathbf{q}_{\parallel}|)/\pi}{(\omega-\omega_{21}-\omega_{\mathbf{k}+\mathbf{q}_{\parallel},\mathbf{k}})^2+\gamma^2(k,\,|\mathbf{k}+\mathbf{q}_{\parallel}|)}\;, \tag{4.5}$$

where $S^{\rm tot}\equiv\int S(\omega)d\omega$ is the total radiation energy density in the direction $\mathbf{q}$. Substituting (4.5) and (4.4) into (4.3), expanding the integral in powers of $\mathbf{q}_{\parallel}$ to within linear terms, and integrating over the orientation angles of $\mathbf{k}$, we obtain the linear current density $J=I/W$ (where W is the device width) in the form

$$J=\frac{2e^3 f_{12}\sin^3\chi\, S^{\rm tot}}{\hbar^3 mc^2}\int_0^{\infty}\frac{f_1(E)\gamma_0(E)dE}{(\Delta\omega)^2+\gamma_0^2(E)}\times\left[-\hbar\tau_2(E)\left[E\frac{\partial\ln\tau_2(E)}{\partial E}+1\right]+\frac{[\tau_1(E)-\tau_2(E)]}{(\Delta\omega)^2+\gamma_0^2(E)}\left[2\Delta\omega-\frac{(\Delta\omega)^2-\gamma_0^2(E)}{2\gamma_0(E)\tau_2(E)}\hbar\frac{\partial\ln\tau_2(E)}{\partial E}\right]E\right]\;, \tag{4.6}$$

where $\Delta\omega\equiv\omega-\omega_{21}$, and γ_0 is defined by (with $E\equiv\hbar^2k^2/2m$)

$$\gamma_0(E)\equiv\gamma(k,k)=\frac{1}{2\tau_1(k)}+\frac{1}{2\tau_2(k)}\;. \tag{4.7}$$

In the derivation of (4.6) it has been assumed that $\Delta\omega\ll\omega_{21}$. Inasmuch as $S^{\rm tot}$ represents the radiation density *inside* the QW, the preceding equations are independent of the chosen illumination scheme. Our waveguide geometry (with the assumed TM polarization of the wave) is described by letting $\chi=\pi/2$ in Eqs. (4.4) and (4.6).

If the momentum-relaxation times are independent of the kinetic energy, then (4.6) simplifies and it can be written in the following form for the three illumination schemes introduced in Sec. II:

$$J_a=S^{\rm tot}\frac{ed\sqrt{\epsilon}\,\alpha(\omega)\overline{\tau}(\omega)}{mc}\;, \tag{4.8a}$$

$$J_b=s_{0,z}^{\rm tot}\frac{e\sin\phi\,\beta_b(\omega)\overline{\tau}(\omega)}{mc}\;, \tag{4.8b}$$

$$J_c=S_0^{\rm tot}\frac{e\sin(2\chi)\sqrt{\epsilon}\,\beta_c(\omega)\overline{\tau}(\omega)}{2mc}\;, \tag{4.8c}$$

where

$$\overline{\tau}(\omega)=-\tau_2+(\tau_1-\tau_2)\frac{2\Delta\omega\langle E\rangle}{\hbar[(\Delta\omega)^2+\gamma_0^2]}\;, \tag{4.9}$$

$\alpha(\omega)$ and $\beta(\omega)$ are the absorption coefficients discussed in the Appendix, and

$$\langle E\rangle\equiv\frac{\int f_1(E)E\,dE}{\int f_1(E)dE} \tag{4.10}$$

is the average electron energy in the ground subband. Equation (4.8a) coincides with the result obtained earlier[19] for a degenerate electron distribution, i.e., when $\langle E\rangle=E_F/2=\pi\hbar^2 n/2m$.

As discussed earlier, selectivity of the excitation with respect to the initial electron momentum $\hbar\mathbf{k}$ results from the conservation laws (3.4). Photons with $\Delta\omega=\omega-\omega_{21}>0$ induce the intersubband transitions only in those electrons whose momentum has a positive component in the direction of $\mathbf{q}_{\parallel}$ [cf. Fig. 2(b)]. Selective promotion of such electrons gives rise to a current within the ground subband—corresponding to the motion of electrons in the direction *opposite* $\mathbf{q}_{\parallel}$. This is the origin of the second, frequency-dependent term in (4.9). This term is positive for $\Delta\omega>0$. At the same time, electrons excited into the subband 2 for $\Delta\omega>0$ have positive velocities in the direction of $\mathbf{q}_{\parallel}$, i.e., their contribution to the current is negative. When the surplus photon energy changes sign, i.e., becomes a "deficit" $\Delta\omega<0$, the current I_1 changes its sign too.

It is clear that for $\langle E\rangle\gg\gamma_0$, the term proportional to $(\tau_1-\tau_2)$ dominates in Eq. (4.9)—except when $\Delta\omega$ is either very small, $|\Delta\omega|\ll\gamma_0$, or very large, $|\Delta\omega|\gg\gamma_0$. The changing of the current direction with varying photon frequency represents a signature of the photon-drag current in a two-dimensional electron gas.

Consider a numerical example of an $Al_xGa_{1-x}As/GaAs/Al_xGa_{1-x}As$ QW with a modulation-doped 2DEG of density $n=10^{12}$ cm^{-2}. Assume $\epsilon\approx 10$, $m=0.067m_0$, and $\hbar\omega_{21}=0.12$ eV (corresponding to the incident radiation wavelength $\lambda_0\approx 10$ μm), and the energy-independent relaxation times $\tau_1=10^{-12}$ s and $\tau_2=10^{-13}$ s (corresponding to $\hbar\gamma\sim 3.5$ meV). The oscillator strength of intersubband transitions is close to unity,[21] $f_{12}\approx 1$. To obtain optical transitions for incident wavelengths in the vicinity of $\lambda_0\approx 10$ μm, the QW width d_W must be of order 100 Å. Figure 4 shows the spectral

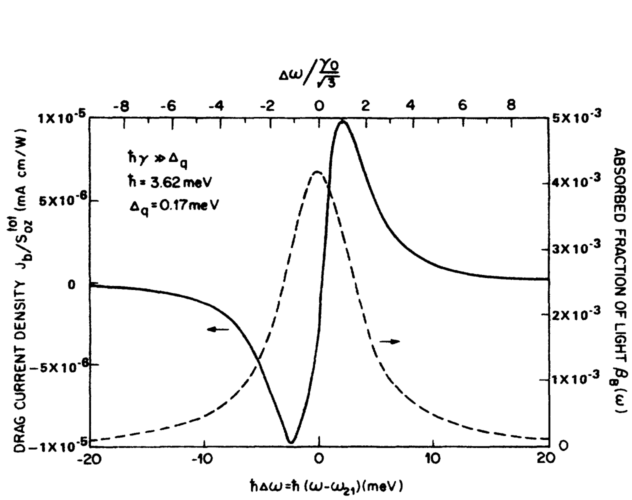

FIG. 4. The calculated spectral dependences $\beta_b(\omega)$ and $J_b(\omega)$ for oblique illumination as in Fig. 3(b) in the limit $\Delta_q\ll\hbar\gamma$.

dependences of the absorption parameter $\beta_b(\omega)$ and the PDE linear current density $J_b(\omega)$, calculated from Eqs. (A3) and (4.8b), respectively, assuming an oblique illumination under Brewster's angle [$\phi=\arctan(\sqrt{\epsilon})\approx 72°$]. It is assumed that the entire surface of the device is illuminated and the current is measured with an ideal short-circuit arrangement (no series resistance). The 2DEG is assumed degenerate. Denoting the QW width by W, the ratio of the full current $J_b W$ to the total incident flux of the radiation energy $S_{0,z}^{\text{tot}} WL$ will exceed the value J_b plotted in Fig. 4 in units of $S_{0,z}^{\text{tot}}$ by the factor $(1/L)$. To obtain $J_c(\omega)/S_0^{\text{tot}}$, one should multiply the values $J_b(\omega)/S_{0,z}^{\text{tot}}$ presented in Fig. 4 by a factor

$$\frac{4\epsilon(1+\epsilon)}{(1+\sqrt{\epsilon})^2}\sin^2(X)\tan X \ .$$

The values of $\beta_c(\omega)$ are related to $\beta_b(\omega)$ plotted in Fig. 4 by

$$\frac{\beta_c(\omega)}{\beta_b(\omega)}=\frac{4\epsilon\sqrt{1+\epsilon}}{(1+\sqrt{\epsilon})^2}\sin(X)\tan X \ ,$$

where X is now the angle between the oblique crystal surface and the QW plane [cf. Fig. 3(c)].

Figure 5 shows the spectral dependences of the PDE current J_a/S^{tot} and the absorption coefficient $\alpha(\omega)$ for the waveguide illumination scheme, calculated with the same QW parameters as those used in Fig. 4. In the waveguide scheme, an important parameter is the confinement factor Γ [cf. Eq. (2.1)]; we have assumed a 25-period superlattice ($p=25$), each period $d=d_W+d_B$ consisting of a $d_W=100$ Å GaAs QW and an AlAs barrier of thickness $d_B=3d_W$, corresponding to the duty cycle $r=d_W/d=\frac{1}{4}$. The assumed permittivity values[26] were $\epsilon_{\text{GaAs}}=10.88$, and $\epsilon_{\text{AlAs}}=8.16$, giving $\epsilon_{\text{core}}-\epsilon_{\text{clad}}\equiv r\Delta\epsilon\approx 0.68$, and $\Gamma\approx 0.18$. The current was calculated with the help of (4.8a) and multiplied by the number of wells. The absorption coefficient was calculated using (A17). If we neglect the extinction of the radiation along the QW (i.e., assume $\alpha L \ll 1$, cf. Sec. V), then, as

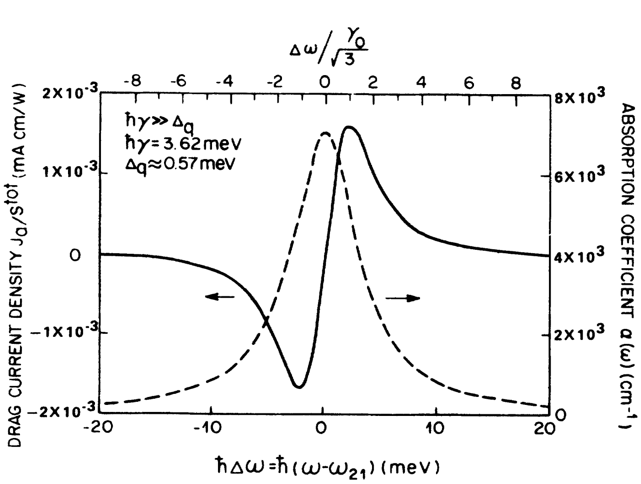

FIG. 5. The calculated spectra of the absorption coefficient $\alpha(\omega)$ and the PDE current density $J_a(\omega)$ for a waveguide geometry in the limit $\Delta_q \ll \hbar\gamma$.

in the preceding, the ratio of the full current to the total radiation energy flux is given by $J/S^{\text{tot}}d$.

For all three illumination schemes, the peaks of the PDE current $|J|$ occur very near [exactly at, if the term τ_2 in (4.9) is neglected] the value $\Delta\omega=\gamma_0/\sqrt{3}$. Thus, the separation between the peaks scales' in proportion to $\gamma_0\approx 1/2\tau_2$, and the signal amplitude *at these peaks* is $\propto \tau_1\tau_2 n$.

V. QUANTUM EFFICIENCY

The quantum efficiency η of a photodetector is defined as the number of electrons flowing in the circuit per incident photon. For example, in the waveguide configuration this means

$$\eta \equiv \frac{J/e}{(S/\hbar\omega)d} \ . \tag{5.1}$$

It may appear from the equations of Sec. IV that for a sufficiently high product $\alpha\bar{\tau}$, the quantum efficiency can exceed unity. This is, of course, not the case. In order to understand this fact more clearly, consider the simplest case of energy-independent τ_1 and τ_2 and rewrite Eqs. (5.1) and (4.8a) in the form

$$\eta=\alpha s\bar{\tau} \ , \tag{5.2}$$

where $s\equiv 2\pi\hbar/\lambda m$ with λ being the photon wavelength in the medium, $\alpha\equiv\alpha(\omega)$, and $\bar{\tau}\equiv\bar{\tau}(\omega)$. Large values of η mean that the quantity $s\bar{\tau}$—which physically corresponds to the longest mean free path of excited carriers—becomes comparable to the absorption length α^{-1}. But since the mean free path cannot exceed the sample length, $s\bar{\tau}<L$ (in a singly connected sample), in this situation we must also have $\alpha^{-1}<L$. However, in this limit, Eq. (5.2) is incorrect because it does not take into account the attenuation of the beam along the QW. It is easy to show that the correct expression, valid for arbitrary values of αL, is of the form

$$\begin{aligned}\eta&=L^{-1}(1-e^{-\alpha L})s\bar{\tau}\\ &\sim \alpha s\bar{\tau} \text{ for } \alpha L \ll 1\\ &\sim L^{-1}s\bar{\tau} \text{ for } \alpha L \gtrsim 1 \ .\end{aligned} \tag{5.3}$$

Since the mean free path in a singly connected sample has to be less than L, it follows that $\eta<1$ in both limits. In principle, $\eta>1$ can be obtained if the sample has the topology of a ring.

VI. DRAG-EFFECT CURRENT IN THE LIMIT OF LOW COLLISIONAL BROADENING $\hbar\gamma \ll \Delta_q$

If we should ignore the photon momentum, then in the limit of a vanishing collision width $\gamma\to 0$, the absorption-spectrum line between two ideal equidistant (i.e., neglecting nonparabolicity) subbands would represent a sharp resonance peak, limited (in the dipole approximation) by a spectral source width $\Delta\omega$. Taking into account the photon momentum, however, leads to a broadening of the line by an amount of order $\Delta_q/\hbar\approx\hbar k_F q_\parallel/m$; as discussed earlier, this broadening is a near-perfect analogy to the Doppler width of spectral lines in gases.

To evaluate the drag current in this case, using our general formalism (4.3) and (4.4), we replace $S(\omega)$ in Eq. (4.4) by $S^{\text{tot}}\delta(\omega-\omega_{21}-\omega_{\mathbf{k}+\mathbf{q}_{\parallel},\mathbf{k}})$. From (4.3) we then obtain

$$J=S^{\text{tot}}\frac{e^3 v_0 f_{12}\{\tau\}}{c\hbar^2\epsilon^{3/2}\omega_{21}} , \tag{6.1}$$

where

$$\{\tau\}=\frac{mc^2}{\hbar\omega}\frac{\Delta\omega}{\omega}\langle\langle \tau_1-\tau_2^*\rangle\rangle-\epsilon\sin^2\chi\langle\langle\tau_2^*\rangle\rangle , \tag{6.2}$$

$$\tau_2^*\equiv\tau_2(E_{\mathbf{k}}+\hbar\Delta\omega) , \tag{6.3}$$

and $\langle\langle\ldots\rangle\rangle$ denotes a special kind of averaging of the quantities $\tau(E_{\mathbf{k}})$ over the momental of electrons in the 2DEG, [cf Eq. (A25)] of the Appendix; the quantity v_0 (dimensionality of a velocity) represents a normalization factor [cf. Eq. (A19)] involved in that averaging. If the relaxation time within a subband can be considered independent of the energy, then one should replace $\langle\langle\tau_i\rangle\rangle$ by τ_i.

Specializing to our three illumination schemes (Sec. II), Eq. (6.1) yields

$$J_a=S^{\text{tot}}\frac{ed\alpha(\omega)\{\tau\}}{mc\sqrt{\epsilon}} , \tag{6.4a}$$

$$J_b=S_{0,z}^{\text{tot}}\frac{e\beta_b(\omega)\{\tau\}}{mc\sin\phi} , \tag{6.4b}$$

$$J_c=S_0^{\text{tot}}\frac{e\beta_c(\omega)\{\tau\}}{mc\tan\chi} . \tag{6.4c}$$

The absorption parameters $\alpha(\omega)$, $\beta_b(\omega)$, and $\beta_c(\omega)$ in the Doppler linewidth limit are given in the Appendix by Eqs. (A18), (A21), and (A22), respectively.

Calculated spectral dependences of the linear current density J_b and the coefficient β_b, are presented in Fig. 6. To ensure the inequality $\hbar\gamma\ll\Delta_q$, the relaxation times τ_1 and τ_2 were assumed equal to 4×10^{-11} and 10^{-11} s, respectively. This value of τ_1 corresponds to an electron mobility 10^6 cm^2/V s—a value often achieved and exceeded in modulation-doped QW's at low temperatures. In the calculation we assumed one QW with the parameters—other than the relaxation times—identical to those assumed in the calculation of curves in Fig. 4. As before, a degenerate 2DEG was assumed. The current peaks occur very near $\Delta\omega=\Delta_q/\hbar\sqrt{2}$; if the second term in (6.2) is neglected, then this position of the peaks is exact.

Figure 7 shows the corresponding spectral curves for the linear current density J_a and the absorption coefficient α in the waveguide scheme, calculated with the help of Eqs. (6.4a), (A18), and (A23). As in the calculation of Fig. 5, we assumed a 25-period multiple QW structure and a degenerate 2DEG ($T=0$).

We see that the curves calculated in the Doppler limit (Sec. VI) have a qualitatively different shape compared to those calculated in the collision-broadened limit (Sec. IV). The main quantitative difference is in the position of the peaks of drag-current spectra: In the Doppler limit these peaks scale with $\Delta_q\propto q_{\parallel}\sqrt{n}$, whereas in the collision-broadened limit they scale with $\gamma_0\approx1/(2\tau_2)$.

VII. SUMMARY AND DISCUSSION

We have developed a detailed theory of the photon-drag effect in intersubband absorption by a two-dimensional electron gas in an inversion layer or a quantum well. If the momentum of incident radiation has a component $\mathbf{q}_{\parallel}$ within the plane of the 2DEG, then the latter acquires, in the process of intersubband absorption, a nonvanishing drift velocity which manifests itself as a measurable drag current. As discussed earlier,[16,19] an important enhancement of this effect occurs when the momentum-relaxation times, τ_1 and τ_2, in the ground and the excited subbands are different, since in this case the

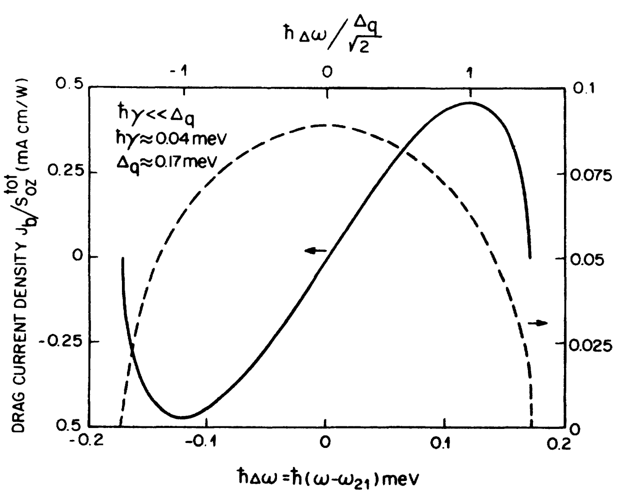

FIG. 6. The calculated spectra $\beta_b(\omega)$ and $J_b(\omega)$ for oblique illumination as in Fig. 3(b) in the limit of low collision broadening, $\hbar\gamma\ll\Delta_q$, and zero temperature.

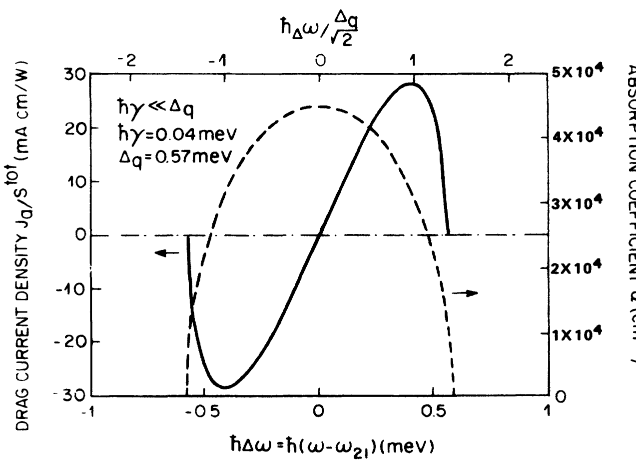

FIG. 7. The calculated spectra of the absorption coefficient $\alpha(\omega)$ and the PDE current density $J_a(\omega)$ for a waveguide geometry in the limit of low collision broadening, $\hbar\gamma\ll\Delta_q$, and zero temperature.

average velocity imparted to the 2DEG electrons can exceed $\hbar q_\parallel$ by several orders of magnitude. The enhanced effect arises due to the conservation of energy and quasimomentum in an elementary act of intersubband absorption, which leads to a velocity-selective excitation of electrons (the Doppler effect) and a violation of the cylindrical symmetry in the electron-momentum distribution function in each of the two subbands. It is not important that the difference $\tau_1-\tau_2$ be large compared to τ_2; on the contrary, for best results one should maximize both relaxation times. As discussed in the Introduction, the effect has important analogies in three-dimensional systems. On one hand, it is conceptually similar to the PDE in bulk *p*-type germanium in the case when the excitation energy is resonant with optical phonons.[14,15] On the other hand, the PDE due to transitions between parallel bands can be regarded as an analog of the light-induced drift of atoms in a gas phase.[18]

We have considered the photon-drag effect in two limits, corresponding to two dominant mechanisms of line broadening in the intersubband absorption spectra. One limit, discussed in Sec. VI and considered earlier by Dykhne *et al.*,[16] corresponds to an idealized situation when one can neglect the collision broadening compared to the "natural" (Doppler) linewidth. In order to realize this limit, one would have to "turn off" the lifetime broadening of the upper subband associated with phonon emission. This may become possible if the intersubband separation is below the optical phonon threshold, $\hbar\omega_{21}<\hbar\omega_{\rm opt}\approx 35$ meV; in other words, one has to be in the range of far-infrared absorption, $\lambda_0 \gtrsim 40$ μm. In this case, the mobility in *both* subbands may approach $\mu\sim 10^6$ cm/V s, corresponding to τ of order 40 ps. The other—more realistic—limit (considered in Sec. IV) corresponds to the dominating lifetime width. Compared to our earlier treatment,[19] we have extended the discussion to the case of energy-dependent relaxation times $\tau_i(E)$ in each subband.

In both limits, a particular attention has been paid to the geometrical aspects of a PDE experiment. Besides the waveguide geometry discussed earlier, we considered two schemes for oblique illumination of the sample. Such schemes offer a considerable practical advantage by permitting experimental structures with a single QW (it would be very difficult to couple light in a waveguide with such a narrow core). Considering the fundamental limit on the quantum efficiency η of a photon-drag detector (defined here as the number of electrons flowing through a closed circuit per incident photon), we showed that in a singly-connected sample η cannot exceed unity (as a matter of principle, this limitation may be lifted if the sample has the topology of a ring).

Let us now discuss the possibility of an experimental observation of the enhanced PDE. As shown in Secs. IV and VI, the signature of such an effect is the changing of the current direction with varying photon frequency. In our view, the only practical difficulty is associated with an *inhomogeneous* broadening of the intersubband absorption line. Indeed, throughout this work we have assumed that the quantum wells are perfectly uniform over the entire sample. If, however, the QW width varies, then so does the intersubband frequency ω_{21}. At a given excitation frequency ω, the quantity $\Delta\omega\equiv\omega-\omega_{21}$ would then be positive in one part of the sample and negative in another. Fluctuations of the well width $d\approx 100$ Å, even by one atomic layer ($\delta d\approx 5$ Å), can destroy the enhanced effect since one would have, approximately,

$$\frac{\delta\omega_{21}}{\omega_{21}}\sim\frac{2\delta d}{d}\approx 10\% \ .$$

This difficulty, however, does not appear insurmountable. Atomically flat heterointerfaces can be assured by interrupted-growth epitaxial techniques[27,28] over relatively large areas (several μm). Steps which go in tandem—without affecting the QW width—should not be a problem. Also, part of the nonuniformity can be expected to be screened away by a large-density 2DEG. For an observation of the enhanced PDE, it is imperative that the inhomogeneous broadening be less than the lifetime width $\hbar\gamma$ considered in this work.

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APPENDIX A: ABSORPTION COEFFICIENT AND ABSORPTION STRENGTH

For the oblique illumination schemes, we define an absorbed-fraction coefficient β as the ratio of the radiation energy at frequency ω, absorbed in the QW, to the normal component of the total radiation energy incident on the sample. For the geometry corresponding to Fig. 3(b), this coefficient can be written in the form

$$\beta_b(\omega)=-\frac{2\hbar\omega}{S_{0,z}^{\rm tot}LW}\sum_{\mathbf{k}}\left[\frac{\partial f_1(\mathbf{k})}{\partial t}\right]_{\rm phot} . \tag{A1}$$

In the limit $\Delta_{\rm q}\ll\hbar\gamma$, substituting $[\partial f_1(\mathbf{k})/\partial t]_{\rm phot}$ from (4.4) and $S(\omega)$ from (4.5) into (A1), we find

$$\beta_b(\omega)=\frac{2e^2f_{12}}{c\hbar^2}F(\phi)\int\frac{\gamma_0(k)f_1^0(E)dE}{(\Delta\omega)^2+\gamma_0^2(k)} , \tag{A2}$$

where $E=\hbar^2k^2/2m$, and

$$F(\phi)=\frac{2\sin\phi\sin(2\phi)}{\epsilon(\sqrt{\epsilon}\cos\phi+\cos\chi)^2} . \tag{A3}$$

For γ_0 independent of the energy, we have

$$\beta_b(\omega)=\frac{2\pi^2ne^2f_{12}}{mc}\frac{\gamma_0/\pi}{(\Delta\omega)^2+\gamma_0^2}F(\phi)\equiv\frac{\bar\beta_b}{\hbar}\frac{\gamma_0/\pi}{(\Delta\omega)^2+\gamma_0^2} . \tag{A4}$$

The absorption strength, defined by

$$\bar\beta_b\equiv\int\beta_b(\omega)d(\hbar\omega) ,$$

is thus given by

$$\bar\beta_b=\frac{2\pi^2e^2\hbar f_{12}}{mc}nF(\phi) . \tag{A5}$$

It is convenient to express the quantities $\beta_b(\omega)$ and $\bar{\beta}_b$ in terms of their values obtained when the light is incident under Brewster's angle (as was the case in recent intersubband absorption experiments[21–23]). In this case, the function (A3) is of the form $F(\phi)=1/(\epsilon\sqrt{1+\epsilon})$, and we have

$$\beta_b(\omega)=\beta_b^{\text{Brewster}}(\omega)\,\epsilon\sqrt{1+\epsilon}\,F(\phi) \tag{A6}$$

and

$$\bar{\beta}_b=\bar{\beta}_b^{\text{Brewster}}\,\epsilon\sqrt{1+\epsilon}\,F(\phi)\ . \tag{A7}$$

For the geometry of Fig. 3(c), the coefficient β_c, defined earlier, can be written in the form

$$\beta_c(\omega)=-\frac{2\hbar\omega}{S_0^{\text{tot}}LL_0}\sum_{\mathbf{k}}\left[\frac{\partial f_1(\mathbf{k})}{\partial t}\right]_{\text{phot}}, \tag{A8}$$

where the length L_0 is indicated in Fig. 3(c), and L determines, as in the other scheme, the length of an illuminated portion of the QW. These lengths are related by $L_0=L\cos\chi$. Since in the present case $|\mathbf{S}(\omega)|$ and $|\mathbf{S}_0(\omega)|$ are connected by (2.5), we can determine $\beta_c(\omega)$ by substituting $S(\omega)$ from (4.5) and $(\partial f_1/\partial t)_{\text{phot}}$ from (4.4) into (A8)

$$\beta_c(\omega)=\frac{8e^2 f_{12}\sin^2\chi}{c\hbar^2(\sqrt{\epsilon}+1)^2\cos\chi}\int\frac{\gamma_0(k)f_1^0(E)dE}{(\Delta\omega)^2+\gamma_0^2(k)}\ . \tag{A9}$$

For γ_0 independent of the energy, Eq. (A9) yields

$$\beta_c(\omega)=\frac{8\pi^2 ne^2 f_{12}}{mc(\sqrt{\epsilon}+1)^2}\frac{\sin^2\chi}{\cos\chi}\frac{\gamma_0/\pi}{(\Delta\omega)^2+\gamma_0^2}F(\phi)$$
$$\equiv\frac{\bar{\beta}_c}{\hbar}\frac{\gamma_0/\pi}{(\Delta\omega)^2+\gamma_0^2}\ , \tag{A10}$$

where the absorption strength $\bar{\beta}_c$ is defined by

$$\bar{\beta}_c\equiv\int\beta_c(\omega)d(\hbar\omega)=\frac{8\pi^2 e^2 f_{12}\hbar n}{mc}\frac{\sin^2\chi}{\cos\chi(\sqrt{\epsilon}+1)^2}\ . \tag{A11}$$

The values of β_c can also be expressed in terms of $\beta_b^{\text{Brewster}}$

$$\beta_c(\omega)=\beta_b^{\text{Brewster}}(\omega)\frac{4\epsilon\sqrt{1+\epsilon}}{(1+\sqrt{\epsilon})^2}\ , \tag{A12}$$

$$\bar{\beta}_c=\bar{\beta}_b^{\text{Brewster}}\frac{4\epsilon\sqrt{1+\epsilon}}{(1+\sqrt{\epsilon})^2}\ . \tag{A13}$$

For a waveguide geometry, the absorption coefficient $\alpha(\omega)$ is defined as the ratio of the total number of optical transitions per unit time to the photon flux density, i.e.,

$$\alpha(\omega)=-\frac{2\Gamma\hbar\omega}{S^{\text{tot}}L^2 d}\sum_{\mathbf{k}}\left[\frac{\partial f_1(\mathbf{k})}{\partial t}\right]_{\text{phot}}\ . \tag{A14}$$

Setting $\chi=\pi/2$ in (4.4), we find for $\Delta_{\mathbf{q}}\ll\hbar\gamma$

$$\alpha(\omega)=\frac{2e^2 f_{12}\Gamma}{cd\hbar^2\sqrt{\epsilon}}\int\frac{\gamma_0(k)f_1^0(E)dE}{(\Delta\omega)^2+\gamma_0^2(k)}\ . \tag{A15}$$

Defining an integrated quantity $\bar{\alpha}\equiv\int\alpha(\omega)d(\hbar\omega)$, analogous to $\bar{\beta}_{b,c}$, we find

$$\bar{\alpha}=\frac{2\pi^2 ne^2\hbar f_{12}}{mcd\sqrt{\epsilon}}\ . \tag{A16}$$

For γ_0 independent of the energy, we have

$$\alpha(\omega)=\frac{2\pi ne^2 f_{12}\Gamma}{mcd\sqrt{\epsilon}}\frac{\gamma_0}{(\Delta\omega)^2+(\gamma_0)^2}\ . \tag{A17}$$

In the case when $\hbar\gamma\ll\Delta_{\mathbf{q}}$, i.e., when the absorption linewidth is of the Doppler nature, the absorption coefficient can be obtained from (A14) by replacing $S(\omega)$ in expression (4.4) for $[\partial f_1(\mathbf{k})/\partial t]_{\text{phot}}$, with $S^{\text{tot}}\delta(\omega-\omega_{21}-\omega_{\mathbf{k}+\mathbf{q}_\parallel,\mathbf{k}})$. The result is

$$\alpha(\omega)=\frac{v_0(\omega)f_{12}\Gamma}{\omega a_0 d}\ , \tag{A18}$$

where $a_0=\epsilon\hbar^2/me^2$ is the effective Bohr radius, and

$$v_0(\omega)\equiv\int_{E_Q}^{\infty}\frac{f_1(E)dE}{\sqrt{[2m(E-E_Q)]}}\ , \tag{A19}$$

with

$$E_Q\equiv\frac{\hbar^2 Q^2}{2m},\quad Q\equiv\frac{m}{\hbar q_\parallel}\Delta\omega\ ;$$
$$E_Q=\left[\frac{\Delta\omega}{\omega}\right]^2\frac{mc^2}{2\epsilon\sin^2\chi}\ . \tag{A20}$$

Similarly, the absorbed-fraction coefficients $\beta_{b,c}(\omega)$ are of the form

$$\beta_b(\omega)=\frac{e^2 m v_0(\omega)f_{12}}{\hbar^2\omega_{21}}\frac{F(\phi)}{\sin\phi}\ , \tag{A21}$$

$$\beta_c(\omega)=\frac{4e^2 m v_0(\omega)f_{12}\tan\chi}{\hbar^2\omega_{21}(\sqrt{\epsilon}+1)^2\epsilon^{1/2}}\ . \tag{A22}$$

The normalization factor v_0 has the dimensions of an effective velocity. For a degenerate distribution this factor equals

$$v_0(\omega)=\Theta(E_F-E_Q)\sqrt{2(E_F-E_Q)/m}\ , \tag{A23}$$

[where $\Theta(x)$ is the step function], whereas for a Maxwellian distribution v_0 is given by

$$v_0(\omega)=\frac{\pi^{3/2}\hbar^2 n}{2^{1/2}m^{3/2}(kT)^{1/2}}e^{-E_Q/kT}\ . \tag{A24}$$

With the help of the effective velocity v_0, the relaxation times, "averaged" over the electron energy in a given subband, are expressed as follows:

$$\langle\langle\tau(E)\rangle\rangle\equiv\frac{1}{v_0(\omega)}\int_{E_Q}^{\infty}\frac{f_1(E)\tau(E)dE}{\sqrt{2m(E-E_Q)}}\ . \tag{A25}$$

[1]H. E. M. Barlow, Proc. IRE **46**, 1411 (1958).
[2]L. E. Gurevich and A. A. Rumiantsev, Fiz. Tverd. Tela (Leningrad) **9**, 75 (1967) [Sov. Phys.—Solid State **9**, 55 (1967)].
[3]N. A. Brynskikh, A. A. Grinberg, and E. Z. Imamov, Fiz. Tekh. Poluprovodn. **5**, 1735 (1971) [Sov. Phys.—Semicond. **5**, 1516 (1972)].
[4]A. M. Danishevskii, A. A. Kastalskii, S. M. Ryvkin, and I. D. Yaroshetskii, Zh. Eksp. Teor. Fiz. **58**, 544 (1970) [Sov. Phys.—JETP **31**, 292 (1979)].
[5]A. F. Gibson, M. F. Kimmitt, and A. C. Walker, Appl. Phys. Lett. **17**, 75 (1970).
[6]V. G. Agafonov, P. M. Valov, B. S. Ryvkin, and I. D. Yaroshetskii, Fiz. Tekh. Poluprovodn. **6**, 909 (1972) [Sov. Phys.—Semicond. **6**, 783 (1972)].
[7]A. A. Grinberg, D. S. Bulianitsa, and E. Z. Imamov, Fiz. Tekh. Poluprovodn. **7**, 45 (1973) [Sov. Phys.—Semicond. **7**, 29 (1973)].
[8]V. G. Agafonov, P. M. Valov, B. S. Ryvkin, and I. D. Yaroshetskii, Fiz. Tekh. Poluprovodn. **6**, 2219 (1972) [Sov. Phys.—Semicond. **6**, 1868 (1973)].
[9]A. A. Grinberg and L. L. Makovskii, Fiz. Tekh. Poluprovodn. **4**, 1162 (1970) [Sov. Phys.—Semicond. **4**, 981 (1970)].
[10]P. M. Valov, B. S. Ryvkin, S. M. Ryvkin, E. V. Titova, and I. D. Yaroshetskii, Fiz. Tekh. Poluprovodn. **6**, 123 (1972) [Sov. Phys.—Semicond. **6**, 99 (1972)].
[11]A. A. Grinberg, N. A. Brynskikh, and E. Z. Imamov, Fiz. Tekh. Poluprovodn. **5**, 148 (1971) [Sov. Phys.—Semicond. **5**, 124 (1971)].
[12]A. C. Walker and D. R. Tilley, J. Phys. C **4**, L376 (1971).
[13]P. M. Valov, B. S. Ryvkin, S. M. Ryvkin, and I. D. Yaroshetskii, Phys. Status Solidi B **53**, 65 (1972).
[14]A. A. Grinberg and L. V. Udod, Fiz. Tekh. Poluprovodn. **8**, 1012 (1974) [Sov. Phys.—Semicond. **8**, 658 (1974)].
[15]A. A. Grinberg, E. D. Belorusets, and E. Z. Imamov, Fiz. Tekh. Poluprovodn. **5**, 2010 (1971) [Sov. Phys.—Semicond. **5**, 748 (1972)].
[16]A. M. Dykhne, V. A. Roslyakov, and A. N. Starostin, Dok. Akad. Nauk. SSSR. **254**, 599 (1980) [Sov. Phys. Dokl. **25**, 741 (1980)].
[17]F. Kh. Gel'mukhanov and A. M. Shalagin, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 773 (1979) [JETP Lett. **29**, 711 (1979)]; E. M. Skok and A. M. Shalagin, *ibid.* **32**, 201 (1980) [*ibid.* **32**, 185 (1980)].
[18]References to recent experimental and theoretical work on the light-induced drift in the gas phase can be found in the work by F. Kh. Gel'mukhanov, J. E. M. Haverkort, S. W. M. Borst, and J. P. Woerdman, Phys. Rev. A **36**, 164 (1987).
[19]S. Luryi, Phys. Rev. Lett. **58**, 2263 (1987).
[20]M. Born and E. Wolf, *Principles of Optics,* 6th ed. (Pergamon, Oxford, 1980), Secs. 1.5 and 2.4.
[21]L. C. West and S. J. Eglash, Appl. Phys. Lett. **46**, 1156 (1985).
[22]B. F. Levine, R. J. Malik, J. Walker, K. K. Choi, C. G. Bethea, D. A. Kleinman, and J. M. Vandenberg, Appl. Phys. Lett. **50**, 273 (1987).
[23]B. F. Levine, K. K. Choi, C. G. Bethea, J. Walker, and R. J. Malik, Appl. Phys. Lett. **50**, 1092 (1987).
[24]A. A. Grinberg, Zh. Eksp. Teor. Fiz. **58**, 989 (1970) [Sov. Phys.—JETP **31**, 531 (1970)].
[25]M. Goeppert-Mayer, Ann. Phys. (Leipzig) **9**, 273 (1931).
[26]J. P. Leburton and K. Hess, J. Vac. Sci. Technol. **B1**, 415 (1983).
[27]H. Sakaki, M. Tanaka, and J. Yoshino, Jpn. J. Appl. Phys. **24**, L417 (1985).
[28]Hiroyuki Sakaki (private communication).