Dynamic Early Effect in Heterojunction Bipolar Transistors

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Abstract—**A theory of the base transport in an abrupt-junction heterostructure bipolar transistor (HBT) is developed in the diffusion limit. The theory is valid for a continuous range of emitter injection energies Δ and accounts for the Early effect (EE) both in the static and the high-frequency limits. Small-signal network parameters strongly depend on Δ and differ from those in a graded emitter–base junction HBT.**

The Early effect (EE) [1] in bipolar junction transistors (BJT's) corresponds to the modulation of the emitter (J_E) and the collector (J_C) currents by the collector–base voltage V_{BC}. The effect arises from the variation of the base width W with V_{BC} and manifests itself in both static and high-frequency characteristics [2]. The theory of the EE in heterostructure bipolar transistors (HBT's), as presented in the recent literature (see for instance [3]), applies only to the case of a graded-gap emitter–base junction, where it is identical to the classical BJT theory. Since the effect increases for shorter W at a given base doping level, it is sometimes stated [4] that EE may be of importance in epitaxial HBT, where the base is narrow. However, modern HBT's are usually designed with an abrupt heterojunction spike [5] (see Fig. 1). For a sufficiently large conduction-band discontinuity Δ (such that $e^{-\Delta/kT} \ll D/v_T W \approx l_{sc}/W$, where $v_T \equiv (kT/2\pi m)^{1/2}$ is the thermionic velocity, D is the electron diffusivity, and l_{sc} is the electron scattering length in the base), the emitter current is controlled [6]–[10] by the emitter–base voltage V_{BE} and does not depend on the base width. Evidently, in this limit there can be no Early effect on the emitter current. In the collector characteristics in the static limit the EE may appear only by virtue of recombination in the base. The present letter develops a simple theory of the base transport in an abrupt-junction HBT, which is valid for arbitrary values of Δ and gives an adequate account of the Early effect both in the static and high-frequency limits.

The base is assumed uniformly doped and has a constant bandgap. We consider the case of a sufficiently large base width, $W \gg D/v_T$, so that the base transport is adequately described by the diffusion current $J(z) = eD\,dn/dz$, where n is the electron concentration. The diffusion equation for n can be split into two equations describing the steady-state concentration $\bar{n}$ and the small-signal oscillatory part $\hat{n}$:

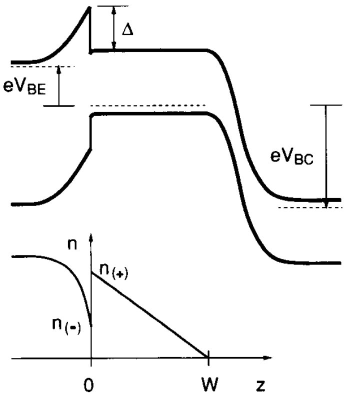

Fig. 1. Band diagram of an abrupt-junction n-p-n HBT and a schematic electron concentration profile.

$$d^2\bar{n}/dz^2 - \lambda_D^2 \bar{n} = 0 \tag{1a}$$

$$d^2\hat{n}/dz^2 - \lambda^2 \hat{n} = 0 \tag{1b}$$

where $\lambda = \lambda_D(1 + i\omega\tau_{cp})^{1/2}$ and $\lambda_D = (D\tau_{cp})^{-1/2}$ with τ_{cp} being the recombination time of electrons in the base. Here and below we are using the phasor notation, $A(t) \equiv \bar{A} + \hat{A}e^{j\omega t}$, for harmonically varying quantities $A(t)$.

The electron concentration suffers a discontinuity at the abrupt heterointerface. Denote the values of n on the two sides of this interface by $n_{(-)}$ and $n_{(+)}$. These two concentrations are not in equilibrium. The former, $n_{(-)} \propto e^{-eV_{BE}/kT}$, gives rise to the thermionic flux of emitter electrons while the latter forms an oppositely directed thermionic flux. Clearly, $n_{(+)}$ is the boundary value for (1), but this value is not known *a priori*. The emitter current is given by the difference of the two fluxes: $J_E = -en_{(-)}v_T + en_{(+)}v_T e^{-\Delta/kT}$. On the other hand, the same current in the base equals $eD\partial n/\partial z$. Therefore, the boundary condition at the emitter interface can be written in the form:

$$\left.\frac{\partial n}{\partial z}\right|_{z=0} = -\frac{v_T}{D}\left(n_{(-)} - n_{(+)}e^{-\Delta/kT}\right). \tag{2}$$

Conditions for the validity of (2) in the static regime [10] coincide with those for the thermionic theory of a Schottky diode. At high frequencies, (2) is valid provided $\omega\tau_\varepsilon \ll 1$, where τ_ε is the electron energy relaxation time.

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At the collector interface, where in the absence of EE the concentration is usually set to zero,[1] the effect of base width modulation by $\hat{V}_{BC}$ can be replaced by the boundary condition of a modulated concentration:

$$\hat{n}(W) = \bar{n}(W - \hat{W}) - \bar{n}(W) = -\hat{W}\left.\frac{\partial \bar{n}}{\partial z}\right|_{z=W} = -\frac{\bar{J}_C \hat{W}}{eD} \tag{3}$$

where $\hat{W} = W\hat{V}_{BC}/V_A$ and V_A is the Early voltage.

The stationary equation (1a) and the small-signal equation (1b) are, respectively, subject to the following boundary conditions:

$$\left(\bar{n}e^{-\Delta/kT} - \frac{D}{v_T}\frac{d\bar{n}}{dz}\right)_{z=0} = \bar{n}_{(-)}, \qquad \bar{n}(W) = 0 \tag{4a}$$

$$\left(\hat{n}e^{-\Delta/kT} - \frac{D}{v_T}\frac{d\hat{n}}{dz}\right)_{z=0} = \hat{n}_{(-)}, \qquad \hat{n}(W) = -\frac{\bar{J}_C \hat{W}}{eD}. \tag{4b}$$

Solutions of these equations are of the form:

$$\bar{n}(z) = \frac{v_T \bar{n}_{(-)}}{D\lambda_D} \frac{\sinh[\lambda_D(W - z)]}{\cosh(\lambda_D W)[1 + \xi(\lambda_D, z)]} \tag{5a}$$

$$\hat{n}(z) = \frac{v_T \hat{n}_{(-)}}{D\lambda} \frac{\sinh[\lambda(W - z)]}{\cosh(\lambda W)[1 + \xi(\lambda, W)]}$$

$$-\frac{\bar{J}_C \hat{W}}{eD} \frac{1 + \xi(\lambda, z)}{1 + \xi(\lambda, W)} \frac{\cosh(\lambda z)}{\cosh(\lambda W)} \tag{5b}$$

where the function $\xi(\lambda, z)$ is defined by

$$\xi(\lambda, z) \equiv \frac{v_T}{\lambda D} e^{-\Delta/kT} \tanh(\lambda z). \tag{6}$$

Below we shall denote $\xi(\lambda, W)$ by ξ_λ and $\xi(\lambda_D, W)$ by ξ_D.

Common-base characteristics are readily obtained from (5). In what follows, the sign of J_C is changed to conform to the network notation. Equation (5a) yields expressions for the steady-state currents in terms of $\bar{n}_{(-)}$:

$$\bar{J}_E = -\frac{e\bar{n}_{(-)}v_T}{1 + \xi_D}; \quad \bar{J}_C = \frac{e\bar{n}_{(-)}v_T}{(1 + \xi_D)\cosh(\lambda_D W)} \tag{7}$$

and (5b) the small-signal currents:

$$\hat{J}_E = \hat{V}_{BE} \frac{g_E(1 + \xi_D)}{1 + \xi_\lambda}$$

$$- \hat{V}_{BC} \frac{g_A \lambda_D W \xi_D}{(1 + \xi_\lambda)\sinh(\lambda_D W)\cosh(\lambda W)} \tag{8a}$$

$$\hat{J}_C = -\hat{V}_{BE} \frac{g_E(1 + \xi_D)}{(1 + \xi_\lambda)\cosh(\lambda W)}$$

$$+ \hat{V}_{BC} \frac{g_A \lambda W[\tanh(\lambda W) + \xi_\lambda \coth(\lambda W)]}{(1 + \xi_\lambda)\cosh(\lambda_D W)} \tag{8b}$$

where $g_E \equiv e|\bar{J}_E|/kT$ is the differential emitter conductance and $g_A \equiv |\bar{J}_E|/V_A$ is a conductance defined in terms of the Early voltage V_A.

Equations (8a) and (8b) are valid for hetero- as well as for homojunction transistors. Assume for simplicity the case of low recombination:

$$\lambda_D W \ll 1, \quad \xi_D = \frac{v_T W}{D} e^{-\Delta/kT} \tag{9}$$

and consider the limits. For sufficiently large Δ, we have ξ_D, $\xi_\lambda \ll 1$ and (8a) and (8b) reduce to

$$\hat{J}_E = g_E \hat{V}_{BE} \tag{10a}$$

$$\hat{J}_C = -g_E \hat{V}_{BE} \operatorname{sech}(\lambda W) + g_A \hat{V}_{BC} \lambda W \tanh(\lambda W). \tag{10b}$$

Equations (10a) and (10b) describe what may be termed the proper HBT limit at high frequencies. In the opposite limit, $\xi_D \gg 1$, which occurs at relatively low Δ, we have

$$\hat{J}_E = g_E \hat{V}_{BE} \frac{\lambda W}{\tanh(\lambda W)} - g_A \hat{V}_{BC} \frac{\lambda W}{\sinh(\lambda W)} \tag{11a}$$

$$\hat{J}_C = -g_E \hat{V}_{BE} \frac{\lambda W}{\sinh(\lambda W)} + g_A \hat{V}_{BC} \frac{\lambda W}{\tanh(\lambda W)}. \tag{11b}$$

Equations (11a) and (11b) describe the BJT limit. They agree with those presented by Tiwari [3] for the case of a graded-gap HBT and are identical to the standard model of a homojunction transistor [2]. Physically, in the limit $\xi_D \gg 1$, the net electron flux, (2), turns out to be much lower than either of its two "thermionic" components, so that the concentrations on the two sides of the abrupt junction are in approximate equilibrium, $n_{(-)} \approx n_{(+)}e^{-\Delta/kT}$.

Equations (8a) and (8b) give a general expression for the common-base intrinsic admittance matrix $y^{(b)}$ of an abrupt-heterojunction HBT in the diffusion approximation. Normally, one is interested in the regime $|\lambda W| \ll 1$, corresponding to frequencies f below f_T. In this case, parameters $y_i^{(b)}$ and $y_f^{(b)}$ are similar in both the HBT and the BJT limits. However, the two components controlled by the Early effect, $y_r^{(b)}$ and $y_o^{(b)}$, are quite different. Relative to the BJT case, the output admittance $y_o^{(b)}$ is

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[1] Within the validity range of the diffusion approximation, $l_{sc}/W \ll 1$, it is permissible to neglect $n(W)$ even for a finite carrier velocity $v = v_s$ in the collector space-charge region. Since $v_s \geq v_T$, it is easy to estimate that $n(W) \approx n(0)/(1 + v_s W/D) \leq n(0)l_{sc}/W \ll n(0)$.

multiplied in the HBT limit by a factor $\tanh^2(\lambda W) \approx i\omega W^2/D$, which corresponds to a phase shift by $\pi/2$ and a reduction in magnitude by approximately f/f_T. The reverse admittance $y_r^{(b)}$ strictly vanishes, which expresses the fact that in the HBT limit there is no Early effect, static or dynamic, in the emitter current.

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