Improve Charging Capability for Wireless Rechargeable Sensor Networks using Resonant Repeaters

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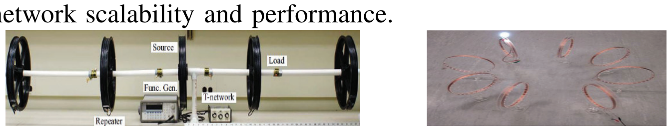
***Abstract*—Wireless charging has provided a convenient alternative to renew sensors' energy in wireless sensor networks. Due to physical limitations, previous works have only considered recharging a single node at a time, which has limited efficiency and scalability. Recent advance on multi-hop wireless charging is gaining momentum to provide fundamental support to address this problem. However, existing single-node charging designs do not consider and cannot take advantage of such opportunities. In this paper, we propose a new framework to enable multi-hop wireless charging using resonant repeaters. First, we present a realistic model that accounts for detailed physical factors to calculate charging efficiencies. Second, to achieve balance between energy efficiency and data latency, we propose a hybrid data gathering strategy that combines static and mobile data gathering to overcome their respective drawbacks and provide theoretical analysis. Then we formulate multi-hop recharge schedule into a bi-objective NP-hard optimization problem. We propose a two-step approximation algorithm that first finds the minimum charging cost and then calculates the charging vehicles' moving costs with bounded approximation ratios. Finally, upon discovering more room to reduce the total system cost, we develop a post-optimization algorithm that iteratively adds more stopping locations for charging vehicles to further improve the results. Our extensive simulations show that the proposed algorithms can handle dynamic energy demands effectively, and can cover at least three times of nodes and reduce service interruption time by an order of magnitude compared to the single-node charging scheme.**

***Index Terms*—Wireless sensor networks, multi-hop wireless charging, resonant repeater, mobile energy replenishment, mobile data gathering, hybrid data gathering.**

I. Introduction

Wireless power transfer has been recently exploited in battery-powered wireless sensor networks (WSNs) to extend network lifetime towards perpetual operations. For high charging efficiency, charging vehicles (denoted as "SenCars" henceforth) are employed to approach sensor nodes in close proximity [1]–[4], [8], [14] and we refer these networks as *wireless rechargeable sensor networks* (WRSNs). However, because the charging efficiency decays as an inverse cube of distance, most of the previous works only considered "short-range" charging where a SenCar needs to approach nodes in very close proximity and can only recharge the nodes one by one. This may lead to extremely long recharging latency: If a rechargeable battery takes 1-4 hours to fully recharge, a network of hundreds of nodes can take days or weeks. During such long latencies some nodes may exhaust energy and cause service interruption. Inspired by the latest advances in mid-range wireless charging (where mid-range refers to energy transmitting distances much larger than the diameter of coils) that can relay energy over several hops to simultaneously replenish multiple nodes, we explore how to leverage this technology to enhance network scalability and performance.



(a) Distribute 15mW energy to 6 loads by 4 repeaters over 2m. Repeater coils are twisted on the black wheels with loads separated in between (courtesy of [13]).

(b) Power a 14W lamp by organizing repeaters into domino form (courtesy of [15]).

Fig. 1. Experimental prototypes of multi-hop wireless charging using resonant repeaters [13], [15].

One of the most cost-effective means to relay energy is *resonant repeaters* (see Fig. 1). They can be easily manufactured from copper coils at low costs. In [12], significant improvements (from 10% to 46%) in efficiency are reported by adding resonant repeaters between the source and receiving coils. In [13], distributing 15mW energy over a distance of 2m to 6 different loads through 4 resonant repeaters has been demonstrated. In [15], experiments have shown that resonant repeaters can be organized into a domino form to power a 14W lamp. Their theoretical results indicate up to 50-70% charging efficiency even after 5-6 hops of relays.

For WRSNs, only very few works have considered recharging nodes in multi-hops [5], [11]. Although pioneering first steps, these schemes do not consider the physics laws governing wireless charging efficiency. It is not only impacted by the distance, vehicle's position, but also by a series of phenomenons such as cross-coupling where complicated interactions between neighboring resonant repeaters cannot be simply ignored. Further, unlike data flows whose rates can be continuously adjusted, an energy flow can be turned on/off but there is no easy means to alter its rate over links [15]. Thus these solutions would deviate from real operating conditions in the network.

To tackle these limitations, in this paper we propose a new multi-hop wireless charging framework to improve charging capability and scalability. With a low-cost repeating circuit installed, sensor nodes can relay energy to their neighbors. Since previous single-node recharge scheduling algorithms do not consider such energy relaying, we provide a new recharge scheduling algorithm for this fundamentally different charging model. Furthermore, energy replenishment has to be considered together with energy consumption patterns, which depend largely on how data are collected. Mobile data gathering reduces energy consumption on intermediate nodes but incurs extra delivery latency [6]–[10], whereas static data gathering has shorter latency but much higher energy costs on routing paths [3]. To achieve a reasonable balance between latency and energy consumption, we introduce a *hybrid data gathering* strategy, where *time-sensitive* data are directly forwarded to the base station and *time-insensitive* data are gathered by mobile collectors.

The new framework raises several interesting questions. First, how to quantify the improvements from charging capability compared to the single-node recharge in terms of the number of nodes a SenCar can cover, and the number of SenCars needed? Second, given time-varying recharge requests, where SenCars should stop to recharge surrounding nodes such that multi-hop wireless charging cost is minimized and how to schedule the SenCars to minimize the moving cost ? Third, are there any relationships between the two types of costs and is there a way to minimize the total system cost? Finally, what is the tradeoff compared to the single-node recharge scheme in energy efficiency, network scalability and packet latency?

To answer these questions, we first show how to accurately calculate wireless charging efficiency based on well-established methods in physics and electronics [15], so as to estimate energy charging cost during multi-hop relay. Then we theoretically analyze the energy consumptions given the hybrid data gathering model and estimate improvements using multi-hop charging. Based on the mathematical model, we can derive the number of

SenCars needed. Further, to minimize both charging and moving costs, we formulate the recharge scheduling problem into the category of location-routing problems [20] with two objectives, respectively. Since the problem is NP-hard, we propose a two-step approximation algorithm that guarantees all energy demands are satisfied while minimizing the costs. In the first step, we identify a set of representative sensor locations (called "anchors") where SenCars stop and recharge nearby nodes such that overall charging cost is minimized. Our algorithm achieves a bounded approximation ratio of $\log n$ to the optimal solution (where n is number of nodes). In the second step, we first utilize approximation algorithms from the Traveling Salesmen Problem to compute a complete shortest recharge tour through anchors. Then we assign recharge routes for different SenCars by dividing the complete tour according to SenCars' recharge capacity, energy demands and multi-hop charging cost. Given the selection of anchors, our algorithm generates recharge tours with the moving cost on SenCars bounded by $(\frac{5}{2} - \frac{1}{2k})$ ratio to the optimal result (where k is number of tours). Finally, upon discovering more room exists to optimize the system cost (charging cost plus moving cost), we propose a post-optimization algorithm that iteratively changes nodes with low charging efficiency into anchors and inserts them back into the established routes to further reduce the overall system cost.

We summarize the contributions of this paper as follows. First, we adopt resonant repeaters to improve charging capability based on realistic modeling of charging efficiency under physics laws. Second, we introduce a hybrid data gathering strategy to achieve balance between routing cost and data latency, and theoretically study scalability improvements. Third, we formulate recharge scheduling into a bi-objective optimization problem and propose a two-step approximation algorithm with bounded approximation ratios for each objective. We discover subtle relations between cost objectives and propose a post-optimization approach to further optimize the system cost. Our evaluation shows that the post-optimization algorithm can reduce the system cost by an additional 25% and the proposed framework can cover more than 3 times of nodes and has significantly less service interruptions compared to previous works. We also demonstrate possible trade-offs between multi-hop and single-node recharge methods. To the best of our knowledge, this is the first work on multi-hop wireless charging for WRSNs based on realistic physics models.

The rest of the paper is organized as follows. Section II outlines the network model, and briefly describes how to compute charging efficiencies. Section III provides theoretical analysis of the framework. Section IV formalizes multi-hop recharge scheduling into a bi-objective optimization problem and proposes a two-step approximation algorithm with a post-optimization algorithm shown in Section V. Section VI provides simulation results. Finally, Section VII discusses limitations and Section VIII concludes the paper.

II. Preliminaries

In this section, we introduce the network model and briefly describe the procedures to calculate multi-hop charging efficiency while bringing comprehensive factors such as mutual inductance and cross-coupling into consideration.

A. Network Model

Fig. 2 shows the basic components in our framework. We assume N sensor nodes are uniformly randomly distributed in a circular field with radius R_c and node density $\rho = \frac{N}{\pi R_c^2}$. An embedded resonant repeater is added into the charging circuitry

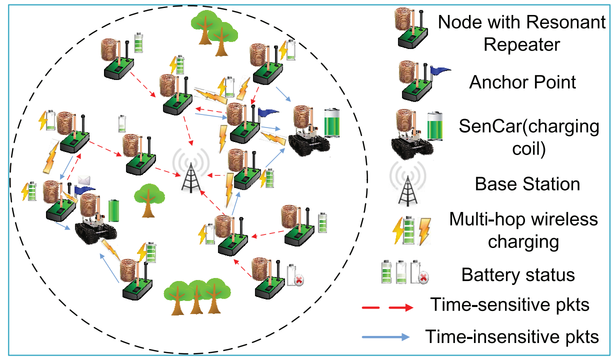


Fig. 2. Multi-hop wireless charging based on resonant repeaters.

on both SenCars and sensors. In contrast to the previous works which have ignored important physics phenomena such as mutual inductance and cross-coupling between neighboring sensor coils, our framework accounts for these factors. For simplicity, we assume all the nodes and SenCars have identical coils with n_t rounds and r_s radius. To successfully relay energy, nodes need to tune their resonant frequencies to the same as SenCars and these nodes form a *charging set* around the stopping location of a SenCar. We can have different resonant frequency bands in the network to avoid interference between neighboring charging sets. Each sensor has a Ni-MH AAA battery with C_s capacity and its recharge time follows the data sheets in [17] (maximum recharge time $T_r = 78$ mins). To provide an effective charge that can stimulate enough currents on sensors' reception circuits, the charging efficiency η should be at least τ, e.g., $\tau = 30\%$; otherwise, the node cannot be charged and it stops relaying wireless energy by switching off its repeating circuitry.

We consider multi-task sensing applications that sensors not only report *time-insensitive* data samples (e.g., temperature and humidity) from the environment periodically, but also detect N_e random events (e.g., lightening strike and tornado warning) that are *time-sensitive*. In a time slot, an event appears independently randomly from other events at a location. With sensing range R_s, an event is detected with probability $p = \frac{R_s^2}{R_c^2}$ for the node in a time slot. We assume sensors generate data packets following a Poisson distribution with rate λ, a common model for events randomly and mutually independently distributed [18]. The energy consumed for transmitting/receiving a packet of length l_p, denoted by e_c, is modeled as in [27], i.e., $e_c = (e_1 d_r^\alpha + e_0) l_p$, where e_1 is the loss coefficient per bit, α is the path loss exponent (usually from 2 to 4) and e_0 is energy consumed on sensing, coding and modulations (we use $e_0 = 50 \times 10^{-6}$ J/bit, $e_1 = 10 \times 10^{-7}$ J/bit, $\alpha = 4$ and $l_p = 32$ bits). A hybrid data gathering strategy is used in our framework to achieve a balance between packet latency and energy consumption. The *time-sensitive* data packets (with rate λ_1) due to event detection are directly forwarded towards the *base station* over multiple hops, while *time-insensitive* data packets (with rate λ_2) are gathered by SenCars during recharge to reduce routing cost.

If a node's battery level falls below threshold β, a recharge request is triggered and sent to SenCars. m SenCars respond to recharge requests cooperatively. Each SenCar is equipped with a more powerful high-capacity battery pack of capacity C_h and consumes e_s J/m energy while moving. SenCars stop at selected sensor locations (called *anchors*) to recharge nodes with multi-hop energy relay and simultaneously gather data packets within l communication hops. If a node is within l-hops of SenCars, it always sends data to the closest SenCar to save energy on intermediate nodes. Nodes generate packets following a Poisson process with rate λ. To ensure packets gathered by SenCars have bounded latency, we impose a recharge time threshold T_d which is the maximum time duration between a SenCar leaving and returning to the base station. If either a recharge tour is finished (i.e., all the recharge requests have been fulfilled) or the timing threshold is reached, the SenCar will return to the base station to upload data packets. T_d indicates the number of charging sets a SenCar can recharge in a tour. For example, if every charging set needs T_r recharge time, at most $\lfloor \frac{T_d}{T_r} \rfloor$ anchors can be covered. When a SenCar is about to deplete its own battery, it returns to the base station for a quick battery replacement. To make better use of SenCar's battery energy, we assume T_d is larger than the recharge time to replenish SenCar's C_h energy into the network. Thus, we can prevent SenCars from traveling back and forth between recharging locations and the base station too frequently for packet uploading.

B. Calculate Multi-hop Wireless Charging Efficiency

Accurate multi-hop wireless charging efficiency is the key in our framework. We describe an approach to calculating efficiency η_n after n relays. In principle, efficiency is governed by *mutual inductance*. Let L_{ij} denote the mutual inductance between repeaters on nodes i and j [16],

$$L_{ij} = \kappa_{ij}(n_t L_s)^2 \approx \frac{r_s^3}{2d_{ij}^3}(n_t L_s)^2 \tag{1}$$

where n_t is the number of rounds of coil wires, κ_{ij} is the magnetic *coupling coefficient* between nodes i and j ($0 \leq \kappa_{ij} \leq 1$), and L_s is the self-inductance of coils[1]. The approximation is taken when wireless charging distance d_{ij} between i and j is much larger than the dimensions of coil radius r_s. Based on Kirchoff's Voltage Law, a well-established method in [15] can be used to calculate charging efficiency. The input voltage from SenCar's transmitting coil induces currents I_2-I_n on all sensor coils oscillating at frequency w and these values can be obtained by solving n linear equations as shown in [15].

The above computation ensures mutual inductance and cross coupling effects are accounted in our model. Assuming the resistance of the repeater circuit is R, and output load resistance is R_L, the efficiency η_n at the n-th repeater output is,

$$\eta_n = \frac{R_L I_n^2}{\sum_{k=1}^{n} R I_k^2 + R_L I_n^2}.$$

As an example, in Table I, we calculate the charging efficiency for up to 4 hops with $n_t = 300$ rounds and $r_s = 10$ cm coil radius while changing the hop-to-hop distance d from 0.25 m to 1.5 m. First, we can see wireless charging efficiency decreases with more hops. This matches the intuition that energy relay attenuates rapidly from the source. Second, we observe that the efficiency decreases sharply when d is larger. This is because that the mutual inductance declines as an inverse cube of distance. For instance, when $d = 0.25m$, charging efficiency after 4 hop relay (η_4) is still 88%. When $d = 1.5m$, η_2 has reduced to 12% and hardly to provide any effective charge for sensor's battery. Thus, the efficiency depends on the number of intermediate nodes that are relaying energy as well as the distance between them. Based on this method, we can calculate the energy cost during multi-hop charging and compare with previous single-node charging schemes.

[1] $L_s = \mu_0 r_s(\ln \frac{8r_s}{r_d} - 2)$, r_d is the wire radius and μ_0 is the permeability constant equal to $4\pi \times 10^{-7} H \cdot m^{-1}$ (Henry per meter).

TABLE I
CHARGING EFFICIENCY VS. RELAY HOPS

hops	1	2	3	4
$d = 0.25m$	0.99	0.91	0.89	0.88
$d = 0.5m$	0.99	0.68	0.64	0.48
$d = 0.75m$	0.98	0.48	0.46	0.17
$d = 1m$	0.89	0.41	0.34	0.03
$d = 1.25m$	0.68	0.28	0.15	0.01
$d = 1.5m$	0.42	0.12	0.03	0

III. THEORETICAL ANALYSIS OF SCALABILITY

In this section, we theoretically analyze scalability improvements using resonant repeaters and calculate the number of SenCars required to achieve energy balance in the network.

A. Energy Consumptions

First, we analyze the energy consumption for hybrid data gathering which has two types of data destined either for the base station or SenCars. Denote the energy consumed in the network by these two types of data as E_b and E_s, respectively. The total energy consumption E in the network for a time slot is $E = E_b + E_s$. To obtain E_b and E_s, we need to first calculate the proportion of traffic destined for the base station (λ_1) and SenCars (λ_2) from the total traffic rate λ. In each time slot, denote the probability that there is at least one event in a node's sensing range as p_d,

$$p_d = 1 - (1-p)^{N_e} = 1 - \left(1 - \frac{R_s^2}{R_c^2}\right)^{N_e} \tag{2}$$

Because time-sensitive packets are generated after the observation of events with probability p_d, the two types of packets split the original Poisson process with probabilities p_d and $1 - p_d$. By basic probability theory [19], we know that each node generates time-sensitive and time-insensitive packets at rates $\lambda_1 = \lambda p_d$ and $\lambda_2 = \lambda(1 - p_d)$, respectively and we are interested in the means of E_b and E_s.

We assume that the radio coverage has a circular shape with d_r transmission distance so at least $h = \lceil \frac{R_c}{d_r} \rceil$ hops are required to reach the outmost boundary of the sensing circle. We divide the network into h concentric rings where the i-th ring carries all the traffic load from outer rings ($i + 1$ to h). For uniform distribution of node density ρ, the number of nodes in the i-th ring is, $N_i = (2i - 1)d_r^2 \pi \rho$. Since the sum of Poisson random variables is still Poisson, we can calculate the average traffic rate λ_i of the i-th ring

$$\overline{\lambda_i} = (N_i e_c + 2\sum_{j=i+1}^{h} N_j e_c)\lambda = (2h^2 - 2i^2 + 2i - 1)\lambda p_d N_1 e_c \tag{3}$$

By the same token, we derive the mean of E_b by taking the sum of λ_i from the 1st to the h-th rings

$$\overline{E_b} = \sum_{i=1}^{h}(2h^2 - 2i^2 + 2i - 1)\lambda p_d N_1 e_c = \left(\frac{4}{3}h^3 - \frac{1}{3}h\right)\lambda p_d N_1 e_c \tag{4}$$

For estimating E_s, although the actual moving trajectories of m SenCars are unknown and quite difficult to analyze, we can view the data gathering process as m moving circles with radius ld_r ($l < h$). The total energy consumption for l-hop mobile data gathering can be obtained by replacing h with l and p_d with $(1 - p_d)$ in Eq. (4), yielding $(\frac{4}{3}l^3 - \frac{1}{3}l)\lambda(1-p_d)N_1 e_c$. The best scenario with mobile data gathering is that all the time-insensitive packets generated in a time slot are gathered using l-hop communications. Statistically, each SenCar gathers data from $l^2 N_1$ nodes, thus we can estimate e_s as

$$\overline{E_s} = \left(\frac{4}{3}l^3 - \frac{1}{3}l\right) e_c N_1 \lambda(1-p_d)N/(l^2 N_1) = \left(\frac{4l^2 - 1}{3l}\right)\lambda(1-p_d)N e_c \tag{5}$$

Since the two types of packets are generated independently on different nodes, the total data traffic is still Poisson. Thus by combining Eqs. (4) and (5), we obtain the mean of E as

$$\overline{E} = \big(\frac{4h^2-1}{3h}p_d + \frac{4l^2-1}{3l}(1-p_d)\big)e_c\lambda N \tag{6}$$

B. Energy Replenishment

Our next objective is to calculate the energy replenishment R in a time slot. Since charging efficiency depends on the actual number of sensor nodes that can relay energy, the procedure requires to solve a set of linear equations whose closed form result is difficult to derive. To circumvent these difficulties, we estimate the maximum charging capabilities of SenCars instead.

Assume a maximum charging range $r_m = f(\rho, \tau)$ which is a function of node density ρ and efficiency threshold τ. Due to the uniform node distribution, if all the nodes within r_m request recharge, a maximum of $\pi r_m^2 \rho$ nodes will be replenished simultaneously by a SenCar. Given recharge threshold β $(0 < \beta < 1)$ for the actual number of nodes that are requesting recharge, on average, $\beta\pi r_m^2\rho$ nodes would benefit in each recharge operation. In the worst case, if no node beyond the immediate hop in range r_m requests recharge, the scheme reduces to the conventional single node recharge.

For each recharge, a SenCar replenishes at least $(1-\beta)C_b$ energy for each node so the total energy it can put back into the network is $C_b(\beta-\beta^2)\pi r_m^2\rho$. If SenCars keep replenishing node's battery one after another without any idle time, the only time overhead is the moving time between anchor locations. Since anchors could be anywhere during operations, we use the diameter which is the longest distance in the field as an upper bound on the moving time, $T_m = 2R_c/v$, where v is the speed of a SenCar. Hence, we can write the collectively replenished energy amount from m SenCars,

$$R = \frac{C_b(\beta-\beta^2)\pi r_m^2 \rho m}{(T_m + T_r)}. \tag{7}$$

We can see that multi-hop charging provides a scalability gain to cover $\max(\beta\pi r_m^2\rho, 1)$ more nodes compared to the single node recharge scheme. For example, if $\beta = 0.5$, $r_m = 3m$ and $\rho = 0.5$ nodes/m^2, on average, a SenCar can replenish 7 nodes simultaneously by spending T_r time, thus speeding up 7 times compared to the single node recharge. This shows that given the same number of SenCars, multi-hop charging enjoys much better scalability to support larger networks.

C. Energy Balance

After the expressions for energy consumption and replenishment have been derived, we can set up an energy balance for the network by letting $E = R$. This relation states that in each time slot, the amount of energy consumed by sensor nodes should be equivalently replenished back into the network by the SenCars. Since E is a random variable and R is in the form of m, the number of SenCars, we can derive m which also follows Poisson distribution. After plugging in Eq. (6) and Eq. (7), we have the mean of m,

$$\overline{m} = \frac{\big(\frac{4h^2-1}{3h}p_d + \frac{4l^2-1}{3l}(1-p_d)\big)h^2 d_r^2 e_c \lambda (T_m + T_r)}{C_b(\beta-\beta^2)r_m^2}. \tag{8}$$

Remarks: It is interesting to see from Eq. (8) that using multi-hop wireless charging, for a fixed field size, the number of SenCars no longer depends on the number of sensor nodes in the network. This property has created opportunities to add more nodes into the network without increasing the number of SenCars. In practice, as node redundancies are usually preferred, multi-hop wireless charging helps network administrators improve scalability without incurring extra manufacturing and human labor costs by introducing more charging vehicles.

IV. Scheduling SenCars for Multi-hop Charging

In this section, we study how to schedule m SenCars for multi-hop wireless charging to respond to sensors' energy requests. A variety of practical factors, e.g., location-dependent charging efficiencies, energy charging cost, SenCar's recharge capacity, and energy consumption in movements, are brought into our problem formulation.

Our objectives are two-folds: on one hand, we aim to minimize the energy cost via multi-hop charging. It requires SenCars to select advantageous locations (anchors) for stopping so that overall charging efficiency is maximized. On the other hand, we want to minimize moving energy consumption for SenCars within their recharge capacities. In principle, our problem resembles the location-routing problem (LRP) [20]. LRP finds the optimal warehouse locations for minimum accessing and distributing costs of traversal routes over demand locations that start and end at warehouses. It encompasses two NP-hard problems, i.e., location and routing problems, and seeks to provide an integrated solution to optimize the overall system cost. However, instead of vehicles directly visiting each warehouse location in the original LRP, our problem involves an additional level of cover problem. That is, the anchors have to ensure that all sensors are "covered," i.e., be charged either directly or via multi-hops. Based on the energy requests at different times, SenCars need to calculate anchors and fulfill all requests from sensors adaptively.

Thus we formulate our problem into the context of LRP with two objectives that minimize both energy cost in SenCars' charging and moving cost. Due to the NP-hardness nature of our problem, we propose a two-step approximation algorithm. In the first step, a ratio of $\log n$ to the optimal charging cost is achieved (where n is the total number of recharge requests). In the second step, given the selection of anchors, the maximum touring cost is bounded by a ratio of $(\frac{5}{2} - \frac{1}{2k})$ to the optimal solution, where k is the number of scheduled tours (normally, $k = m$). Finally, based on the results from the algorithm, we study the relationships between the objectives and combine them into a single-objective problem using the weighted method [26]. A post-optimization algorithm is proposed to further reduce the total system cost by inserting anchors into the established routes.

A. Problem Formulation

We now present the formulation of our problem. During operations, energy information from sensor nodes can be gathered by SenCars using the methods in [4]. At time t, given the set of SenCars, $\mathcal{M}$, the set of sensor nodes requesting recharge, $\mathcal{N}$, the set of potential anchors where SenCars can stop, $\mathcal{A}(\mathcal{A} \subseteq \mathcal{N})$, and the set of starting locations of SenCars, $\mathcal{I}$, we formulate the problem as follows.

Consider a graph $G = (V, E)$, where V_i $(i \in \mathcal{N} \bigcup \mathcal{I})$ is the location of sensor node i, and E are edges connecting sensor nodes. The weight of an edge E_{ij} is the energy cost c_{ij} traveling on the edge, which is proportional to the distance between nodes i and j. Each SenCar has recharge capacity C_h corresponding to the maximum number of nodes and distance it can travel in each tour. A node i has energy demand d_i (which equals full capacity minus its residual energy). Each anchor a covers a set of nodes $\mathcal{S}_a$ and the entire covered set of all the anchors achieves $\mathcal{N}$ $(\bigcup \mathcal{S}_{a\in\mathcal{A}} = \mathcal{N})$. Recharging $\mathcal{S}_a$ requires t_a time which is usually determined by the node with the longest recharge time. For a node i, η_{ia} denotes the recharge efficiency when a SenCar resides at anchor a. Several decision variables are introduced in the formulation. x_{ijk} is 1 if anchor $i \in \mathcal{A}$ immediately precedes $j \in \mathcal{A}$ for SenCar k, otherwise it is 0. For $i \in \mathcal{N}, k \in \mathcal{M}, a \in A$, y_{ia} is 1 if node i can be recharged when a SenCar resides at $a \in A$. z_{ik} is 1 if node i is recharged by SenCar k. u_a is 1 if an anchor a is chosen; otherwise, it is 0. v_{ik} is the position of anchor i in the path of SenCar k. Our objective is to minimize the charging cost in multi-hop energy relays, F_c, and SenCars' moving cost, F_m.

$$\mathbf{P1}: \quad \min F = (F_c, F_m) \tag{9}$$

where,

$$F_c = \sum_{i\in\mathcal{N}} \sum_{a\in\mathcal{A}} \frac{1-\eta_{ia}}{\eta_{ia}} d_i y_{ia}, \tag{10}$$

$$F_m = \sum_{i\in\mathcal{A}} \sum_{j\in\mathcal{A}} \sum_{k\in\mathcal{M}} c_{ij} x_{ijk} + \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{A}} \sum_{k\in\mathcal{M}} c_{ij} x_{ijk}. \tag{11}$$

Subject to

$$\sum_{i\in\mathcal{A}} x_{ijk} = z_{jk}, j \in \mathcal{A}, k \in \mathcal{M}, \tag{12}$$

$$\sum_{j\in\mathcal{A}} x_{ijk} = z_{ik}, i \in \mathcal{A}, k \in \mathcal{M}, \tag{13}$$

$$\sum_{a\in\mathcal{A}} y_{ia} = 1, i \in \mathcal{N}, \tag{14}$$

$$\eta_{ia} y_{ia} > \tau, i \in \mathcal{N}, a \in \mathcal{A}, \tag{15}$$

$$y_{ia} \le u_a, i \in \mathcal{N}, a \in \mathcal{A}, \tag{16}$$

$$\sum_{i\in\mathcal{N}} z_{ik} (\sum_{a\in\mathcal{A}} d_i y_{ia}/\eta_{ia}) + \sum_{i\in\mathcal{A}} \sum_{j\in\mathcal{A}} c_{ij} x_{ijk} + \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{A}} c_{ij} x_{ijk} \le C_h, k \in \mathcal{M}, \tag{17}$$

$$\sum_{k\in\mathcal{M}} z_{ak} = u_a, a \in \mathcal{A}, \tag{18}$$

$$\sum_{i\in\mathcal{A}} \sum_{j\in\mathcal{A}} t_i x_{ijk} + \big(\sum_{i\in\mathcal{A}} \sum_{j\in\mathcal{A}} c_{ij} x_{ijk} + \sum_{i\in\mathcal{I}} \sum_{j\in\mathcal{A}} c_{ij} x_{ijk}\big)/v < T_d, k \in \mathcal{M}, \tag{19}$$

$$2 \le v_{ik} \le |\mathcal{N}|, i \in \mathcal{A}, k \in \mathcal{M}, \tag{20}$$

$$v_{ik} - v_{jk} + (|\mathcal{A}| - |\mathcal{M}|) x_{ijk} \le |\mathcal{A}| - |\mathcal{M}| - 1, \quad i, j \in \mathcal{A}, k \in \mathcal{M}, \tag{21}$$

$$x_{ijk}, y_{ia}, z_{ik} \in \{0, 1\}, i, j \in \mathcal{N}, a \in \mathcal{A}, k \in \mathcal{M}. \tag{22}$$

In the above formulation, constraint (12) and constraint (13) stipulate the connectivity of the path that a SenCar stopping at an anchor also leaves it. Constraint (14) imposes that all the nodes request recharge are covered by anchors. Constraint (15) ensures that the recharge efficiency for a node from its anchor should be larger than the efficiency threshold. Constraint (16) guarantees that a node is assigned to one of the anchors. Constraint (17) mandates that the sum of total demands serviced by a SenCar plus its moving energy consumptions should not exceed its recharge capacity. Constraint (18) enforces each anchor visited by only one SenCar. Constraint (19) states that the total time duration of a recharge tour should be within a time threshold T_d. Constraints (20) and (21) are formed according to [21] to prevent subtours of SenCars. Constraint (22) forces x_{ijk}, y_{ia} and z_{ik} to be 0-1 valued.

Remarks: This formulation reflects recharge schedules at time t based on N energy requests (N is an input). For executions at different times, the optimization problem takes corresponding inputs and generates different results (anchors, SenCar schedules, etc).

The above problem is NP-hard because the two subproblems led by the objectives are both NP-hard (Set Cover and Vehicle Routing Problems). Although standard optimization procedures can yield optimal solutions [20], it is prohibitive to run them on SenCars due to enormous computation overhead. The base station has computational resources. However, the communication overhead to maintain updated energy requests and disseminate recharge decisions for the SenCars could be high in a long run. Moreover, the existing optimization methods are usually designed to handle static inputs and lack the flexibility to deal with constant variations in sensor networks such as battery energy and SenCar movements. Therefore, a polynomial-time approximation algorithm with an acceptable bounded ratio is more desirable in practice. To design the approximation algorithm, we follow a natural approach to tackling the objectives sequentially and finally examine the relationships between them. Next, we propose a two-step approximation algorithm which first selects the anchors that minimize energy charging cost, and then finds the minimum recharge routes for SenCars.

B. Approximation Algorithms

In this subsection, we explain the designs of the algorithm. We first define a *charging set* $\mathcal{S}_i$ of a node i as those nearby nodes with charging efficiencies larger than τ when a SenCar stops at node i. At the network initialization phase, each node performs the procedures in Section II-B to compute its effective charging set in a distributed manner. For node i, its neighbor j is included in $\mathcal{S}_i$ only if j's charging efficiency is larger than threshold τ and the corresponding efficiency is denoted as η_j^i ($j \in \mathcal{S}_i$). The algorithm starts with finding the set of anchors.

1) Adaptive Anchor Selection: We define the weight of each set $\mathcal{S}_i$ as the total energy needed to satisfy the recharge demands of these nodes, $w_i = \sum_{j\in\mathcal{S}_i} \frac{(1-\eta_j^i)d_j}{\eta_j^i}$. It is not difficult to observe that our objectives in Eq. (10) is equivalent to minimizing the sum of weights of the selected sets. In general, this problem belongs to the category of original *Set Cover Problem* (SCP) with one difference: While the original SCP allows the results to share the same nodes and thus resultant sets are not necessarily disjoint, our formulation restricts a node to be recharged by only one SenCar (Eq. (18)). Hence in our problem, the resultant sets should be disjoint. We can modify the well-known greedy approach to adapt to the context of our problem.

Our revised greedy algorithm utilizes resonant frequencies to distinguish among charging sets and allows nodes to "join" or "leave" a set very easily by tuning to the proper frequency. Nodes in neighboring charging sets are tuned to different resonant frequencies. The band between different frequencies is wide enough to avoid any interference. The assignment of resonant frequency for each charging set can use existing algorithms (e.g., graph coloring, or minimum frequency span). Due to space limit, we omit the details and a survey of frequency assignment problems can be found in [22].

Initially, we define sets $\mathcal{A}$ and $\mathcal{B}$ to record anchors and their covered node sets respectively and both sets are initialized to empty. First, for each node $i \in \mathcal{N}$, we compute its average weight, $\overline{w_i} = \sum_{j\in\mathcal{S}_i} \frac{(1-\eta_j^i)d_j}{\eta_j^i} / |\mathcal{S}_i|$ and search for the set with the minimum

TABLE II
ADAPTIVE ANCHOR SELECTION ALGORITHM

Input: Recharged node set $\mathcal{N}$, effective charging subset $\mathcal{S}_i$, $i \in \mathcal{N}$, energy demand d_i of node i, charging efficiency of node j η_j^i when a SenCar stops at i, empty sets $\mathcal{A}$, $\mathcal{B}$.
Output: Set of anchors $\mathcal{A}$ and resultant subsets $\mathcal{B}$
While $\mathcal{B} \neq \mathcal{N}$
Calculate $\overline{w_i} = \sum\limits_{j\in\mathcal{S}_i} \frac{(1-\eta_j^i)d_j}{\eta_j^i}/|\mathcal{S}_i|$.
Find minimum weight $k = \arg\min_i \overline{w_i}$, $i \in \mathcal{N}$.
$\mathcal{A} \longleftarrow \mathcal{A} \bigcup k$, $\mathcal{B} \longleftarrow \mathcal{B} \bigcup \mathcal{S}_k$.
$\forall j \in \mathcal{S}_k$, set resonant frequency equal to k.
$\mathcal{S}_i \longleftarrow \mathcal{S}_i - B, \forall i \backslash k \in \mathcal{N}$.
End While

$\overline{w_i}$. Assume node k's subset has the least average weight so k becomes an anchor. Then, it is added into $\mathcal{A}$ and $\mathcal{S}_k$ is put into $\mathcal{B}$ to be marked as "covered." In practice, this is done by tuning all the nodes in $\mathcal{S}_k$ to have the same resonant frequency. Anchor node k can send out packets carrying the frequency information within the boundary of $\mathcal{S}_k$ to achieve this easily. Since other sets have different frequencies, we remove nodes that have already been covered from the remaining sets. Their elements are updated accordingly $\mathcal{S}_i = \mathcal{S}_i - B, \forall i \backslash k \in \mathcal{N}$. At this time, if $\mathcal{B}$ contains all the nodes in $\mathcal{N}$, the algorithm terminates. Otherwise, it continues to find the next set among the remaining nodes with minimum average weight until all the nodes are covered ($\mathcal{B} = \mathcal{N}$). Table II shows the pseudo-code for the anchor selection algorithm.

2) Assign Recharge Routes: After the set of anchors $\mathcal{A}$ has been found, we assign the recharge routes for m SenCars while considering SenCars' capacities along with its moving cost and multi-hop charging cost. Based on [23], we propose an approximation algorithm to bound SenCars' moving energy cost given the anchors. Our approach first utilizes a Traveling Salesman Problem (TSP) algorithm to compute a complete route on $\mathcal{A}$, e.g., 1.5-approximation Christofides algorithm [24]. In this way, we can ensure that anchors close to each other are placed on the same SenCar's recharge route. To facilitate our analysis, we assume the complete tour starts at the base station and ends at the last node for recharge. In fact, the starting positions of SenCars are the ending positions from the last tour and SenCars traverse through the base station to upload data packets. The recharge sequence can be expressed as $r = (b, 1, 2, i, \ldots, n)$, where anchor $i \in \mathcal{A}, n = |\mathcal{A}|$ and b is the base station. To reflect SenCar's starting position, an extra edge with cost $c_{i,b}, i \in \mathcal{I}$ can be added to represent the energy cost from SenCar's starting location $i \in \mathcal{I}$ to the base station b. Let c_{max} denote the maximum energy cost from any node on the path to the base station, $c_{max} = \max\limits_{i\in\mathcal{A}\bigcup\mathcal{I}} c_{b,i}$. The TSP algorithm yields a complete route r that incurs c_r energy cost using one SenCar.

Next, r is split into k tours and k depends on the SenCars' recharge capacity (constraint in Eq. (17)). We check whether an equal division of m SenCars from the total energy cost $(\sum\limits_{i\in\mathcal{A}}\sum\limits_{j\in\mathcal{S}_i} \frac{(1-\eta_j^i)d_j}{\eta_j^i} + c_r - c_{max})/m + 2c_{max}$ is less than SenCar's capacity C_h[2]. If yes, $k = m$. Otherwise, $k = \lceil(\sum\limits_{i\in\mathcal{A}}\sum\limits_{j\in\mathcal{S}_i} \frac{(1-\eta_j^i)d_j}{\eta_j^i} + c_r - c_{max})/(C_h - 2c_{max})\rceil$. We start with an arbitrary direction along r for the partitioning. For each route j, $1 \leq j \leq k$, we search for the last anchor along the complete tour r to ensure the traveling energy cost is no greater than $\frac{j}{k}(c_r - c_{max}) + 2c_{max}$. Let a_i^j and a_l^j represent the i-th and last nodes in the j-th tour, respectively. The j-th tour is obtained as $(\mathcal{I}_j, b, a_1^j, a_2^j, \ldots, a_l^j)$. The pseudo-code of the algorithm is presented in Table III. Next, we prove the approximation bounds for the proposed algorithm.

[2]The term $2c_{max}$ is the maximum energy cost from SenCar's starting position to the base station plus the cost from the base station to the first anchor on the recharge path.

C. Approximation Bounds and Complexity

For $n = |\mathcal{N}|$ recharge requests, our algorithm gives a $\log n$ approximation of the energy cost during multi-hop wireless charging and a $(\frac{5}{2} - \frac{1}{2k})$ ratio for the traveling cost given the selected anchors, where k is the number of tours depending on energy demands and recharge capacity C_h. In the extended greedy algorithm for Set Cover Problem, we assume the optimal energy cost is w^*. During computation, when there are i nodes left to be covered, it incurs at most $\frac{w^*}{i}$ energy cost per node. The bound of the extended greedy algorithm is $\sum\limits_{i=1}^{n} \frac{w^*}{i} = w^* \log n$. The equality holds because the summation $\sum\limits_{i=1}^{n} \frac{1}{i} = \log n$ is the n-th harmonic number.

Next, we prove the traveling energy cost has an approximation ratio of $(\frac{5}{2} - \frac{1}{2k})$ respect to k tours. Here, when $k > m$, the $k-m$ tours are traversed by SenCars after they have replaced batteries in the base station. Nevertheless, the total cost would still be the same. For the complete tour, the energy cost is c_r with the optimal value c_r^*. Use Christofide's minimum spanning tree approximation to the TSP, $\frac{c_r}{c_r^*} \leq 1.5$ [24]. Assume that tour j has the maximum energy cost c_j among k tours and its optimal value is c_j^*. The energy cost for tour j is at most $\frac{1}{k}(c_r - c_{max})$ (excluding the edge leaving the base station in the complete tour r) plus $2c_{max}$ for the two edges connecting the base station to SenCar's starting position and the first anchor in each tour. Therefore, $c_j \leq \frac{1}{k}(c_r - c_{max}) + 2c_{max} = \frac{1}{k}c_r + (2 - \frac{1}{k})c_{max}$. We divide both sides by c_j^*,

$$\frac{c_j}{c_j^*} = \frac{1}{k}\frac{c_r}{c_j^*} + \left(2 - \frac{1}{k}\right)\frac{c_{max}}{c_j^*} \leq \frac{1}{k}k\frac{c_r}{c_r^*} + \left(2 - \frac{1}{k}\right)\frac{1}{2} \leq \frac{5}{2} - \frac{1}{2k} \quad (23)$$

The inequality holds because for each tour, an edge is added to connect the first sensor node to the base station, $c_r^* \leq \sum\limits_{i=1}^{k} c_i^*$. If we divide both sides by k and use the fact that $\max\limits_{1\leq i\leq k}(c_i^*) = c_j^*$, we have $\frac{c_r^*}{k} \leq c_j^*$. We take the approximation $c_{max} \leq \frac{1}{2}c_j^*$. The equality holds when the tour has only one node.

The time complexity for the anchor selection algorithm is $\mathcal{O}(|\mathcal{N}|\log|\mathcal{N}|)$ because finding the anchor with the minimum weight in each step among $|\mathcal{N}|$ nodes requires $\log|\mathcal{N}|$ and there is a total of $|\mathcal{N}|$ iterations. For the recharge route assigning algorithm, in the worst case, there is only one SenCar that needs to recharge $|\mathcal{N}|$ nodes one by one, so the time complexity is $|\mathcal{N}|^3$ dominated by the Christofides algorithm [24].

Remarks: Although the $\log n$ bound for energy charging cost seems quite large, it is essentially one of the best polynomial-time approximation algorithms: it has been proved [25] that the Set Cover Problem cannot be approximated in polynomial time within a ratio of $c \log n$, for $c < \frac{1}{4}$ under general complexity assumptions. A tighter bound might not be necessary given the increased complexity and transient nature of energy requests.

V. POST-OPTIMIZATION BY INSERTING ANCHORS

When the recharge time threshold T_d is not met, there could be further room to optimize results of the two-step algorithm. In

TABLE III
RECHARGE ROUTE ASSIGNING ALGORITHM

Input: Set of anchors $\mathcal{A}$, SenCars $\mathcal{M}$, energy demand d_i of node i, charging efficiency of node j, η_j^i when charger is at i. Set of SenCars' initial locations $\mathcal{I}$, capacity C_h, base station b, max energy cost traveling on an edge c_{max}.
Output: Recharge sequence r_j for SenCar j's tour.
Compute complete TSP recharge path on $\mathcal{A}$ starting from b.
Record the TSP sequence $r = (b, 1, 2, i, \ldots, n)$ with cost c_r.
If $(\sum_{i \in \mathcal{A}} \sum_{j \in \mathcal{S}_i} \frac{(1-\eta_j^i)d_j}{\eta_j^i} + c_r - c_{max})/m + 2c_{max} < C_h$, $k = m$,
Else $k = \lceil (\sum_{i \in \mathcal{A}} \sum_{j \in \mathcal{S}_i} \frac{(1-\eta_j^i)d_j}{\eta_j^i} + c_r - c_{max})/(C_h - 2c_{max}) \rceil$.
Start with an arbitrary direction on r. For tour j, search for the last node a_l^j along r, satisfying $c_j \leq \frac{j}{k}(c_r - c_{max}) + 2c_{max}$.
Obtain j-th tour, $(\mathcal{I}_j, b, a_1^j, a_2^j, \ldots, a_{l_j}^j), 1 \leq j \leq k$.

this subsection, we propose a post-optimization algorithm. Since both objectives in Eq. (10) and Eq. (11) are energy outputs from the SenCar's own battery, we can combine them into a single objective using the weighted method in [26], $F = w_1 F_c + w_2 F_m$. The weights w_1 and w_2 are assigned by network administrators to measure the importance of energy charging cost compared to moving cost. If $w_2 > w_1$, it means that the administrator cares more about SenCar's moving cost over energy charging cost. For example, if $w_2/w_1 = 2$, for total cost F, reducing the moving cost by 1 J is equivalent to saving energy charging cost of 2 J on SenCars. In practice, we would expect $w_2 > w_1$ in most cases as the administrators want to minimize the recharge time by covering more nodes with anchors so a slight increase of energy cost due to multi-hop charging is acceptable.

A. Inserting Anchors

It is critical to observe that the optimal system cost F achieves a good compromise between F_c and F_m. That is, on one hand, introducing more anchors would potentially increase SenCars' moving cost F_m; on the other hand, more anchors means fewer energy relays thus less energy charging cost F_c. Based on this observation, we propose a post-optimization algorithm that evaluates whether inserting an anchor into the established *charging sets* would lead to lower system cost. To keep it simple and effective in a dynamic network environment, we need to avoid computationally intensive algorithms.

The basic procedures is illustrated below. After obtaining a recharge sequence of anchors $(a_1, a_2, \ldots, a_{l_s})$ for SenCar s, it randomly picks one anchor a_i and finds a node j with the maximum charging cost $\max(\frac{1-\eta_{j,a_i}}{\eta_{j,a_i}} d_j)$ in a_i's charging set $\mathcal{S}_{a_i}$. Then we simply designate node j as a new anchor because by charging j directly, a great amount of energy cost can be reduced. We denote node j as a new anchor a'_j. Next, an important step is to see whether we can further reduce energy charging cost by moving some of the elements from $\mathcal{S}_{a_i}$ to $\mathcal{S}_{a'_j}$. This is because a node k in $\mathcal{S}_{a_i}$ may be more efficiently recharged via the new anchor. Therefore, for each node k in $\mathcal{S}_{a_i}$, we compare if

$$\frac{(1-\eta_{k,a_i})}{\eta_{k,a_i}} d_k > \frac{(1-\eta_{k,a'_j})}{\eta_{k,a'_j}} d_k$$

If yes, we move node k to be covered in $\mathcal{S}_{a'_j}$ and denote the old a_i by a'_i after this operation. This is done by tuning the resonant frequency of k to the same as a'_j. After we have examined all the elements, a new anchor a'_j is introduced to potentially split the original charging set while their joint coverage of nodes remains the same.

TABLE IV
POST-OPTIMIZATION ALGORITHM ON SENCAR s

Input: Recharge sequence $a_1, a_2, \ldots, a_{l_s}$ for SenCar s,
Set of anchors $\mathcal{A}_s$, energy demand d_i of node i, charging efficiency of j, $\eta_{j,i}$ if mobile charger is at i,
moving cost $c_{i,j}$ on edge (i, j),
objective weights w_1, w_2, charging set $\mathcal{S}_a$ for all anchors.
Output: A new recharge tour consists of anchors.
While $\mathcal{A}_s \neq \emptyset$
Randomly select an anchor $a_i \in \mathcal{A}_s$.
Find node j with $\max(\frac{1-\eta_{j,a_i}}{\eta_{j,a_i}} d_j), j \in \mathcal{S}_{a_i}$.
Assign j as a new anchor a'_j and $\forall k \in \mathcal{S}_{a_i}$.
If $\frac{(1-\eta_{k,a_i})}{\eta_{k,a_i}} d_k > \frac{(1-\eta_{k,a'_j})}{\eta_{k,a'_j}} d_k$,
$\mathcal{S}_{a'_i} \longleftarrow \mathcal{S}_{a_i} - k$, $\mathcal{S}_{a'_j} \longleftarrow \mathcal{S}_{a'_j} + k$.
$\partial f_m \longleftarrow (c_{a_{i-1},a'_i} + c_{a'_i,a'_j} + c_{a'_j,a_{i+1}}) - (c_{a_{i-1},a_i} + c_{a_i,a_{i+1}})$.
$\partial f_c \longleftarrow \sum_{a \in \{a'_i, a'_j\}} \sum_{k \in \mathcal{S}_a} \frac{(1-\eta_{k,a})d_k}{\eta_{k,a}} - \sum_{k \in \mathcal{S}_{a_i}} \frac{(1-\eta_{k,a_i})d_k}{\eta_{k,a_i}}$.
$\triangle F \longleftarrow w_1 \partial f_c + w_2 \partial f_m$.
If $\triangle F < 0$ and Eq. (19) holds, insert a'_j into route,
$\mathcal{A}_s \longleftarrow \mathcal{A}_s - a_i$.
Else If $\triangle F \geq 0$ and Eq. (19) holds. $\mathcal{A}_s \longleftarrow \mathcal{A}_s - a_i$.
Else Eq. (19) is violated, **Break**.
End While

B. Optimize Total Cost

The next step is to calculate whether there would be a reduction on the total cost F. Denote the changes of moving cost after introducing a'_j by ∂f_m and changes of charging cost by ∂f_c. We assume the new sequence $(a_1, a_2, \ldots, a'_i, a'_j, \ldots, a_{l_s})$ has the lowest moving cost so

$$\partial f_m = (c_{a_{i-1},a'_i} + c_{a'_i,a'_j} + c_{a'_j,a_{i+1}}) - (c_{a_{i-1},a_i} + c_{a_i,a_{i+1}})$$

and

$$\partial f_c = \sum_{a \in \{a'_i, a'_j\}} \sum_{k \in \mathcal{S}_a} \frac{(1-\eta_{k,a})d_k}{\eta_{k,a}} - \sum_{k \in \mathcal{S}_{a_i}} \frac{(1-\eta_{k,a_i})d_k}{\eta_{k,a_i}}$$

Then we see whether $\triangle F = w_1 \partial f_c + w_2 \partial f_m$ is less than zero. If yes, it means a reduction of F is accomplished. We can successfully add new anchor a'_j with its charging set $\mathcal{S}_{a'_j}$ if the additional recharge and traveling time do not violate the latency constraint in Eq. (19). Otherwise, we move on to the next anchor. The algorithm for each SenCar terminates when all the anchors have been checked or a violation of Eq. (19) occurs. The pseudo-code for the post-optimization algorithm is shown in Table IV.

We now analyze its time complexity. Since $|\mathcal{A}|$ anchors are generated from the two-step approximation algorithm, we need to check at most $|\mathcal{A}|$ charging sets. Let us define the size of maximum charging set as $|\mathcal{S}_m|$. Each such iteration requires searching over charging sets of maximum size $|\mathcal{S}_m|$ nodes for the highest charging cost and possibly re-assigns nodes to the new anchor. Hence, the time complexity of the post-optimization algorithm is $\mathcal{O}(|\mathcal{A}||\mathcal{S}_m|)$.

C. An Example of the Algorithm

To see the entire operation of the algorithm more clearly, we show an example in Fig. 3. Fig. 3(a) demonstrates a snapshot during the operation of 3 SenCars ready to resolve 80 recharge requests from nodes with energy demands from 200-1500 J. The first step is to find anchors which offer the entire coverage from all energy requests with the minimal charging cost. Fig. 3(b) shows the results of anchor selection algorithm. 23 anchors are selected and the largest charging set includes 9 nodes. For clarity, we only plot the charging set in Fig. 3(b). In Fig. 3(c), a complete

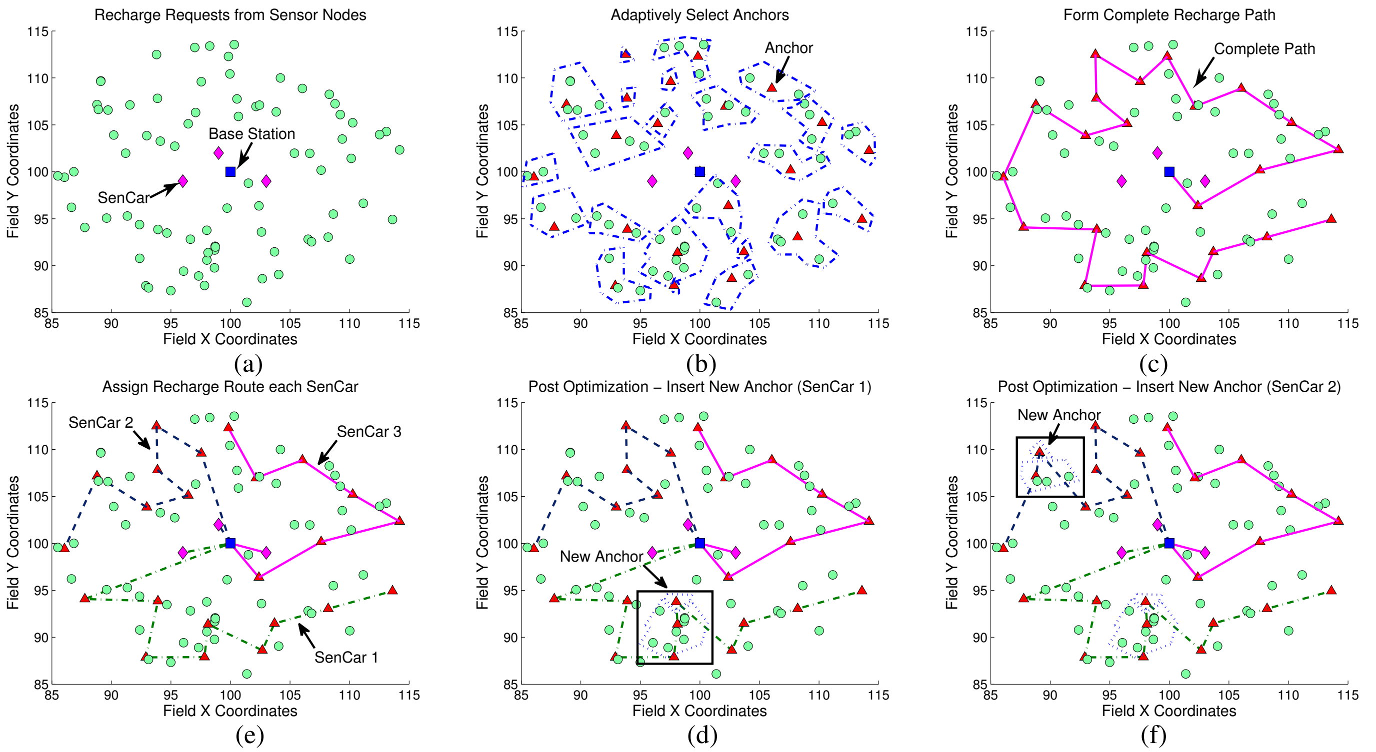


Fig. 3. Example of the algorithm. (a) SenCars receive a number of energy requests. (b) Find anchors among nodes. (c) Form a complete recharge path through anchors. (d) Assign recharge route to each SenCar. (e) Inserting an anchor in SenCar 1's route. (f) Inserting an anchor in SenCar 2's route.

recharge route is found through all the anchors starting from the base station using the Christofides algorithm [24]. In Fig. 3(d), the complete recharge path is split into 3 different routes and each SenCar is assigned a route. Up to this point, SenCars can fulfill all the energy request by stopping at anchor locations and charge nodes in multi-hops.

To further reduce the system cost, we conduct post-optimization procedures for each SenCar. For demonstration purposes, we use weights $w_1 = 1, w_2 = 3$ to evaluate any improvement by inserting an anchor and perform an iteration for all 3 SenCars. A random anchor with its charging set is selected for evaluation in each recharge route. We calculate the value of $\triangle F$ to see whether there is further saving in the system cost. Our algorithm yields $\triangle F_1 = -496$ J for SenCar 1, $\triangle F_2 = -490$ J for SenCar 2 and $\triangle F_3 = 130$ J for SenCar 3. Since $\triangle F_1$,$\triangle F_2$ for SenCar 1 and 2 are less than zero, inserting anchors at the locations shown in Fig. 3(e) has further reductions in system cost. On the other hand, since $\triangle F_3$ for SenCar 3 is larger than zero, there would be a slight increase of the total cost so we should not insert the anchor at the picked set. For clarity, we have shown two successful cases of anchor insertion in Fig. 3(f) for SenCars 1 and 2. The post-optimization process ends after each SenCar has examined all its charging sets for further improvement.

VI. Performance Evaluations

We have developed a discrete-event simulator to evaluate the performance of multi-hop wireless charging (denoted as "MH") and compared it with the conventional scheme where SenCars recharge a single node each time [1]–[4] (denoted as "SN"). We distribute 500 sensor nodes uniformly randomly in a circular field with radius $R_c = 25$m. The transmission distance and sensing range are $d_r = R_s = 5$m. $N_e = 5$ events appear in the field independently randomly. Each time slot is 1 min. Nodes generate packets at an average rate of $\lambda = 3$ pkt/min following Poisson distribution. We use Dijkstra's shortest path routing algorithm to direct packets to their destinations and set the data collection hop l to 2, recharge time threshold T_d to 900 mins.

Once a node's energy falls below the threshold ratio $\beta = 0.5$ to the total capacity, it sends a recharge request to SenCars. We use an AAA NiMH battery of 780 mAh capacity working at 1.5 V. Recharge time is modeled from [17] with a maximum at 78 mins. The MH charging efficiency threshold is $\tau = 0.3$; any node with smaller charging efficiency will not receive any energy. All the SenCars and sensors have identical coils with $n_t = 300$ rounds and $r_s = 10$ cm. Wireless charging efficiencies are calculated using the procedures in Section II-B. Each SenCar is equipped with one 4 Ah battery pack of 12V and moves at 1 m/s ($e_s = 48$ J/m). The simulation is set to run for 6 months' time.

A. Number of Nonfunctional Nodes

We now demonstrate the advantage of MH by comparing the number of nonfunctional nodes with SN. Once a node depletes its battery and no SenCar has arrived yet, it is nonfunctional until being recharged. Fig. 4(a) compares the number of nonfunctional nodes when $N = 500$. To keep nonfunctional nodes within 5%, at least 5 SenCars are needed for SN. In contrast, for MH, only 1 SenCar is needed and 2 SenCars can almost eliminate the chances of battery depletion over the entire operations. The surge of nonfunctional nodes around 10-15 days for SN is because the recharge requests have temporarily exceeded SenCars' capability. As the network reaches equilibrium, the curves decline gradually. However, this phenomenon does not appear with MH which shows better robustness even with fewer SenCars. Recall from Eq. (8), our calculation yields $\overline{m} = 1.06$ which matches our observation here that one SenCar can almost satisfy all the energy requests and two SenCars can maintain nonfunctional nodes close to zero.

To see the scalability improvement more clearly, we have conducted another evaluation where we set the number of SenCars $m = 2$ and the number of nodes $N = 300$ for SN to provide

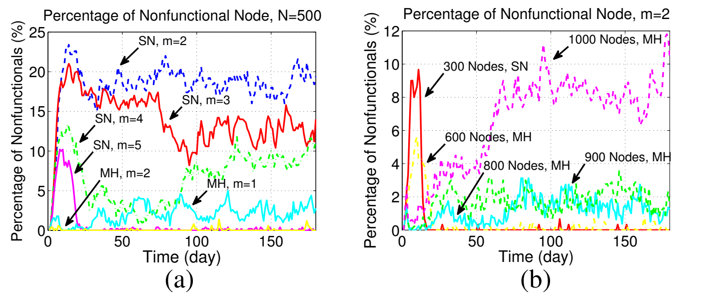


Fig. 4. Comparison on the number of nonfunctional nodes. (a) Performance comparison when $N = 500$. (b) Scalability improvement when $m = 2$.

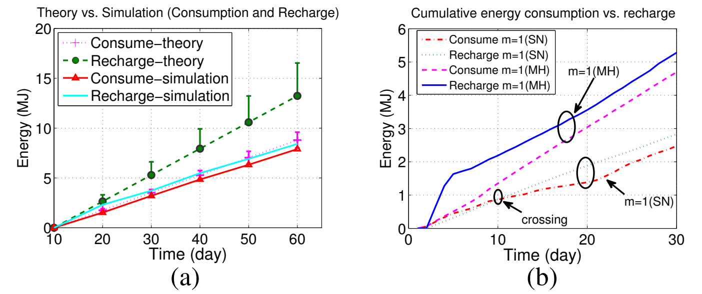


Fig. 5. Energy consumption vs. replenishment $N = 500$. (a) Theoretical results vs. simulations (MH, $m = 1$). (b) Comparison of SN and MH ($m = 1$).

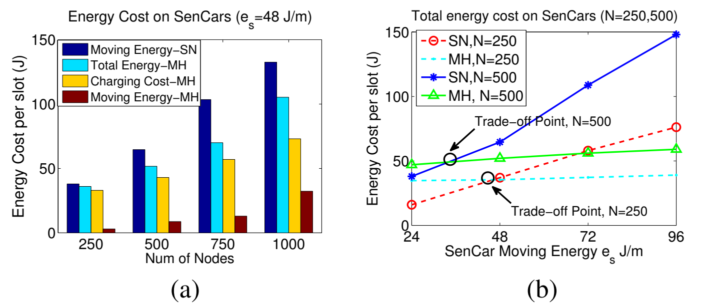


Fig. 6. Comparison of energy cost on SenCars to maintain nonfunctional nodes under 5%. (a) $e_s = 48$ J/m. (b) $e_s = 24$ to 96 J/m.

a baseline and increase N for MH until the nonfunctional percentage rises above 5% (Fig. 4(b)). As we can see, with the same number of SenCars, MH can serve up to 900 nodes (with nonfunctional nodes $< 4\%$), an increase of 3 folds to that of SN.

B. Energy Consumption vs. Replenishment

We evaluate the amounts of energy consumption and replenishment and validate the accuracies of our theoretical model. To better exhibit the gaps between curves, we plot the results for 60 days. Fig. 5(a) depicts energy consumption and replenishment curves for the theoretical, simulation results of MH, $m = 1$. For the theoretical consumption curve, we delineate the mean values with ranges representing standard deviations from the means. For theoretical replenishment curve, we use the slowest charging rate for the battery [17] as a base and a faster rate corresponds to a jump-up indicated by the range of the curve. First, we observe that the replenishment curve is above the energy consumption curve for both theoretical and simulation results. This indicates SenCars can put more energy back into the network than consumed, which is consistent with our results in Fig. 4(a) (almost all the nodes are functional). Our theoretical analysis on the energy consumptions can achieve very high estimation accuracy, as indicated by the small gap between the two curves. The gap between replenishment curves is wider, which is due to the idle time between two successive recharge operations. When the number of SenCars is enough, the recharge requests are sparse over time and SenCars do not need to perform recharge continuously, thus the gap in between.

We also evaluate the cumulative energy evolution in Fig. 5(b). To see the gaps and crossing between curves clearly, we plot the first 30 days of simulation time. For SN, at first, the energy consumption curve is above the replenishment curve until around 10 days when it crosses under the replenishment curve. The crossing marks an important moment as some nodes have depleted their energy and stopped to consume any more energy when the number of SenCars is not enough (corresponding to the surge of 20% nonfunctional nodes in Fig. 5(a)). On the other hand, for MH, the recharge curve is always above the consumption curve and leaves a much wider gap compared to SN, implying a much higher recharge capability from MH.

C. Energy Cost and Trade-offs

An interesting issue is to compare the energy cost of MH to SN and possible trade-off between the two schemes. In Fig. 6, we evaluate the energy cost needed to maintain the same quality of service (nonfunctional $< 5\%$). In Fig. 6(a), for MH, we show energy costs from both node recharging and SenCar traveling, as well as the sum of them and compare with the total cost of SN, while varying N from 250-1000. When $N = 250$, the total cost is almost equivalent while increasing N results in better efficiency for MH. This is because when node density is higher, more nodes can be recharged simultaneously without the hassle to travel to them one by one. If multi-hop charging cost is much less than moving cost e_s, it is more cost-effective to use MH.

To visualize the trade-offs between MH and SN, we adjust the moving cost e_s from 24-96 J/m in Fig. 6(b) which represents different energy efficiencies of the SenCar's battery and motors. For $N = 250$, a trade-off point around 46 J/m is observed. When $e_s < 46$ J/m, SN is more cost-effective. Similar result is observed for $N = 500$ where the trade-off point is around 30 J/m. These results indicate that if energy charging cost can be compensated by moving shorter distances, MH would have less total cost. Based on these results, network administrator can decide which scheme to use given the system parameters.

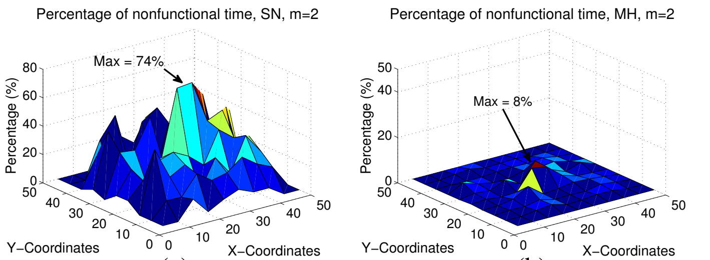
D. Packet Latency and Lengths of Service Interruptions

For successful and timely packet delivery, all the nodes on the routing paths should be functional. If a node becomes nonfunctional on a routing path, its upstream node buffers packets until the routing path is recovered by SenCars. Table V reports average latencies for both time-sensitive (TS) and time-insensitive (TI) packets. We can see MH has much shorter latency than SN for both TS and TI packets because of much lower fractions of nonfunctional nodes during the operations. Once packets are generated, they can be immediately routed to the destination with less chance of experiencing buffering delays.

The time duration while a node is in nonfunctional status greatly impacts the network operation. Such nodes are not able to sense the environment and may miss important events, constituting service interruptions. Fig. 7 shows the percentage of

TABLE V
AVERAGE PACKET LATENCIES (MINS)

	TS(SN)	TI(SN)	TS(MH)	TI(MH)
$m = 1$	692	669	6.24	440
$m = 2$	545	369	6.17	228
$m = 3$	395	249	6.26	162
$m = 4$	246	267	6.21	128
$m = 5$	159	265	5.81	111



(a) (b)

Fig. 7. Comparison of nonfunctional nodes' durations $N = 500, m = 2$. (a) SN. (b) MH.

nonfunctional durations to the entire simulation time. For fair comparison, we set $N = 500$ and $m = 2$ for both cases. SN results a maximum of 74% time in nonfunctional status with the average over 40% widely spreading on the entire field. In sharp contrast, MH has the maximum of only 8% with an average below 3%. This shows that MH has significantly less service interruptions than SN.

VII. Discussions

In practice, the effectiveness of multi-hop wireless charging could be affected by node density and topology. For a sparse network, it is possible a node has no immediate neighbors to relay energy. In this case, our scheme still works, but reduces to a single node recharge method. In reality, nodes are usually deployed at densities much higher than needed for monitoring, which extends lifetime and improves robustness. Topology control mechanisms keep only a small set of nodes working while the rest sleeping to extend network lifetime; a sleeping node can wake up to replace a nonfunctional neighbor to avoid losing sensing coverage. Such high density deployment presents opportunities to apply our multi-hop recharging method: all nodes in certain recharging radius can be waken up to receive energy, and most of them would return to sleep after recharge. How to coordinate duty cycles and recharge schedules is an important issue we plan to investigate next. For sparse networks with low node density, we aim to study how to place more nodes at strategic positions for relaying energy in future.

Another practical challenge is that the node topology may cause misalignment of sensor coils and degrade charging efficiency. Fortunately, recent research using coil arrays provides position-free solutions to the misalignment problem and it is found that charging efficiency increases from 4.8% to 64% [28]. Another option is to use mechanisms similar to "sliding antennas" [29] to fine tune and align the orientations of coils on demand.

VIII. Conclusions

In this paper, we employ resonant repeaters to improve the efficiency and scalability of recharge in WRSNs. We present detailed procedures to calculate multi-hop wireless charging efficiency based on laws in physics and electronics that have been overlooked by previous studies. We introduce a hybrid data collection strategy to achieve a balance between routing cost and data latency, and establish a mathematical model to estimate scalability improvement and the number of SenCars. We formulate the recharge scheduling problem into a multi-objective optimization problem, which is NP-hard. To achieve low-complexity, we propose a two-step approximation algorithm with bounded ratio for each objective followed by a post-optimization algorithm to further reduce the system cost. Finally, we evaluate the proposed framework by extensive simulations and compare with previous works. The results reveal much better network scalability and performance of our algorithm, and also validate our theoretical analysis.

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