Long-term Renewable Energy Usage Maximization in a Microgrid

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***Abstract*—With the development of renewable energy generators and electricity storages, microgrids become a promising technology of the smart grid. Maximizing the usage of renewable energy is vital to running a microgrid as it indicates reduction of the usage of thermal electricity purchased from the macrogrid. To this end, the excessive renewable energy of a user should be transferred to other users who need energy. Unfortunately, coordinating the transfers of renewable energy among the users in the microgrid is particularly difficult due to the stochastic nature of renewable energy, and the dynamic energy demand of each user. In this paper, we consider the problem of maximizing the long-term renewable energy usage by exchanging excessive renewable energy among users in a microgrid. We propose an online control algorithm which determines the amounts of renewable energy transferred among users in an online fashion. We rigorously prove that our online control algorithm is approximately optimal. We have conducted comprehensive simulation results that demonstrate the efficacy of our online algorithm.**

I. Introduction

In recent years, the *smart grid* provides a prospect of a modern network for electricity generation, transmission and consumption, replacing traditional power systems. The smart grid provides two significant advantages. *First*, it employs two-way flows of both electricity and real-time information, facilitating the advanced and intelligent controls on electricity delivery. The backward flow of electricity is enabled by the recent development of distributed generators such as solar panels and small wind turbines [1], which can convert fluctuant and intermittent renewable energy resources into electricity. *Second*, the smart grid uses energy storages which can store excessive energy for future usage. As the technology innovation of rechargeable batteries [2] (e.g., Lithium-ion batteries and flow batteries) continues, more and more smart homes are expected to be equipped with such batteries.

Microgrids are seen as an important component of the smart grid, and have attracted a lot of attention from the industry and governments. A typical microgrid is composed of a central controller and a group of localized electrical consumers (or users) equipped with distributed generators and energy storages as shown in Fig. 1. The information about these users can be automatically delivered to the central controller through wireless or wired communication networks. A microgrid connects to the macrogrid through a single access point, which is controlled by the central controller. When the access point is connected, electricity can be transferred between the macrogrid and the users in the microgrid. When the access point is disconnected, the microgrid enters into the *islanded* mode, where the users are powered by the distributed generators and the storages.

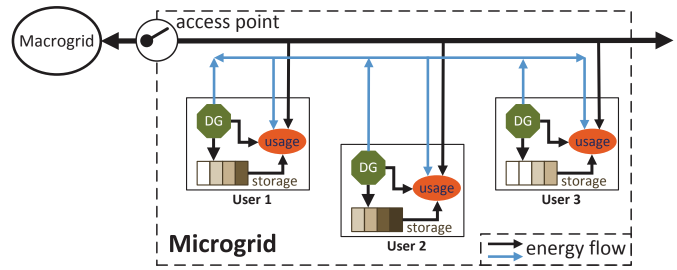

Fig. 1. The illustration of a typical microgrid, which comprises of a number of users sharing a single access point connected to the macrogrid. Electricity is transferred from the macrogrid to users and exchanged among different users.

As distributed generation based on renewable energy resources always achieves lower costs and is more friendly to the environment than traditional thermal power generation, *maximizing the usage of renewable energy is vital to running a microgrid* as it indicates reduction of the usage of thermal electricity purchased from the macrogrid. However, the renewable energy generated in a microgrid may not be fully utilized if each user stays isolated from other users, i.e., the renewable energy generated at a user is only supplied to the same user. The main reason is that the available renewable energy can be excessive compared to the actual demand of the user, and the capacity of the user's energy storage is always limited. To further improve the utilization of renewable energy in a grid, the excessive renewable energy of a user should be transferred to other users who are short of renewable energy.

Unfortunately, coordinating the transfers of renewable energy among the users in the microgrid is particularly difficult. Several major reasons account for the difficulty. *First*, for each user, the excessive energy can be either stored for future use or sold to other users. The decision impacts the amount of energy that the microgrid purchases from the macrogrid. However, it is nontrivial to make the optimal decisions which depend on future energy demands and generated renewable energy. *Second*, the generation of the renewable energy is subject to real-time transient fluctuations of the natural source, e.g., the wind. As a consequence, the amount of future renewable energy generated at a user is typically by nature unpredictable and random. *Third*, the energy demand of a user is also subject to real-time transient changes, which is unpredictable and

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random as well.

A number of approaches [3][4][5] have been proposed for improving energy efficacy, considering fluctuations of renewable energy over time. In [3], Huang et al. assume the basic usage of users in a microgrid can be satisfied by the minimum renewable energy generation. Different with our work, the distributed generators and batteries belong to the microgrid instead of specific users. Renewable energy is allocated to users according to their outage tolerance in quality usage. In [4] and [5], energy exchanging among different microgrids is considered. Both of them focus on minimizing the grid-wide costs, which are incurred by energy transfers between the macrogrid and a microgrid or between two microgrids.

In this paper, we consider the maximization of long-term renewable energy usage through exchanging excessive renewable energy among users in a microgrid. We first formulate the problem of minimizing the total payment given to the macrogrid, which includes the payment for purchasing traditional electricity and the payment for renting the distribution grids to transfer energy, under the requirement of satisfying the dynamic demands of all users. We then propose an online control algorithm which determines for each user the amounts of renewable energy transferred to other users in an online fashion. We rigorously prove that our online control algorithm is approximately optimal. The gap between our algorithm and the optimum is within a tunable constant.

The major contributions are summarized as follows.

- To maximize long-term renewable energy usage in a microgrid, we build models for energy pricing and serving by considering distributed energy generators and storages owned by individual users.
- We propose an online control algorithm for coordinating renewable energy exchange among the users in a microgrid. The online control algorithm achieves provable approximate optimum.
- Both rigorous analysis and comprehensive simulations are conducted to demonstrate the superiority of our control algorithm in comparison to other competing online algorithms.

The remainder of this paper proceeds as follows. The system model including several submodels and the problem formulation are presented in Section II. Section III describes the details of our proposed online control algorithm. Section IV presents the performance of simulations. We discuss related work in Section V. Finally we conclude our paper in Section VI.

II. System Model

A. Overview of a microgrid

In this paper, we consider a typical microgrid system comprising of a number of localized electricity users and a central controller. The *electricity users* can be companies, schools, residential customers and so on. We consider there are N users in the microgrid, denoted by $\mathcal{N} = \{1, 2, \cdots, N\}$. We assume that each user in the microgrid is equipped with a renewable generator and an energy storage, as shown in Fig. 1. The users can be powered by both the macrogrid and the distributed generators.

Energy flows in the microgrid, including energy exchange among users and energy transfer from the macrogrid to users, can be scheduled by the *central controller* [6]. Without loss of generality, we consider controls in the microgrid are made in a time-slotted manner. Each time slot is denoted by $t \in \mathcal{T} = \{1, 2, \cdots, T\}$. The central controller can also communicate with users via wireless or wired networks to collect their information in each time slot.

B. Energy generation and demand

In each time slot, user i harvests an amount of energy from its own generator, $G_i(t)$, and generates electricity usage demand, $D_i(t)$. Due to the uncertainty of the nature of renewable energy sources and the usage of electrical appliances, the energy generation and request of each user is random and unpredictable over time. We assume that both $G_i(t)$ and $D_i(t)$ for $\forall i \in \mathcal{N}$ are independent and identically distributed (i.i.d.) in each time slot. Due to the limitation on the spread of distributed generators at present, we consider the condition that the electricity generated within a microgrid cannot satisfy all its demands over a long period.[1]

Besides the electricity generated by the distributed generators, the macrogrid can provide electricity supply from traditional power plants. These power plants, generating electricity from non-renewable resources such as fossil oil and coal, always have a large energy supply. Thus, the electricity provided by the macrogrid can be seen as infinite in our supply model.

C. Energy pricing model

In our pricing model, we consider two types of prices. One type is used to price the electricity bought from the macrogrid by each user. This kind of prices are decided jointly by power supply companies and the administration of the smart grid. We assume the price paid by user i for a unit of electricity supplied by the macrogrid is $p_i(t)$, which is bounded as $0 < p_i(t) \leq p_i^{max}, \forall i, \forall t$. We assume the price is dynamic over time (e.g., high in peak time) and varies to different users (e.g., large accumulated usage leading to a high price). The other type of prices is used when users rent the electricity distribution grids in the microgrid for personal reasons. For example, users could utilize the existing distribution grids owned by the macrogrid to transmit their renewable energy. We assume the price of renting the distribution grids by user i to transmit a unit of energy is $q_i(t), \forall i, \forall t$. The prices, decided by the administration of the smart grid, may vary according to the congestion condition of electric cables. Intuitively, renting cables to transmit electricity is much cheaper than purchasing the same amount of electricity from the macrogrid.

[1] An individual user may satisfy its own request and have surplus in a short period. For example, a resident goes out for a few days, without electricity consumption in home while the solar panel generates energy continuously.

D. Energy serving model

For each user i, its energy demand $D_i(t)$ should be satisfied in each time slot to maintain the quality of service of smart grids. The energy demand can be met by the generation of its generator, the storage in its battery, transmitting from other users or purchasing from the macrogrid. By comparing its energy generation $G_i(t)$ and demand $D_i(t)$, we consider two cases, where the energy demand of user i is satisfied in different ways:

- If $G_i(t) \geq D_i(t)$, then user i can use its harvested energy to satisfy all its demand in time slot t. Moreover, there is $W_i^+(t)$ energy surplus, where $W_i^+(t) = G_i(t) - D_i(t) \geq 0$. User i can choose to
  1) Store in its battery for future usage. We denote the amount of stored energy as $S_i(t)$, bounded as $0 \leq S_i(t) \leq S_i^{max}$. S_i^{max} is the maximal energy which can be charged to the battery in one time slot.
  2) Transfer the excessive energy to other users in the microgrid who are suffering energy shortages. The amount of energy transferred from user i to user $j (j \neq i)$ in time slot t is represented by $B_{i,j}(t)$.

  Accordingly, we have
$$\sum_{j \neq i} B_{i,j}(t) + S_i(t) \leq W_i^+(t), \forall i \in \mathcal{N}, t \in \mathcal{T}. \quad (1)$$

- If $G_i(t) < D_i(t)$, then user i has $W_i^-(t) = D_i(t) - G_i(t) > 0$ energy demand exceeding its own generation, which can be satisfied by the following three ways:
  1) Release energy stored in its battery to serve the shortage in electricity supply. The released energy $R_i(t)$, bounded by maximal recharging amount R_i^{max}, should be no more than the energy left in the battery (denoted by $B_i(t)$) in the current time slot. Thus, we have
$$0 \leq R_i(t) \leq \min\{R_i^{max}, B_i(t)\}, \forall i \in \mathcal{N}, t \in \mathcal{T}. \quad (2)$$
  2) Obtain energy from other users who have surplus (e.g., $G_j(t) > D_j(t), j \neq i$). Note that we limit that user i can only be served by the renewable energy generated by user j in the current time slot, other than the energy in the battery of j. We assume the energy obtained from user $j (j \neq i)$ is $B_{j,i}(t)$.
  3) Purchase energy from the macrogrid. If the above two ways still do not meet the demand, user i has to buy electricity from the macrogrid as much as needed. The electricity from the macrogrid purchased by user i in time slot t is denoted by $E_i(t)$.

  To satisfy the demand of user i in time slot t by the three ways, we have
$$R_i(t) + \sum_{j \neq i} B_{j,i}(t) + E_i(t) = W_i^-(t) \quad (3)$$

E. Energy storage model

In our storage model, we consider each user has a rechargeable battery, where the energy left in time slot t is denoted by $B_i(t)$. The maximum capacity of the battery of user i is denoted by B_i^{max}. Thus, there is the following constraint,
$$0 \leq B_i(t) \leq B_i^{max}, \forall i, \forall t. \quad (4)$$

Given the amount of recharged energy $S_i(t)$ and discharged energy $R_i(t)$ in time slot t, the energy available in time slot $t+1$ can be updated according to the following rule,
$$B_i(t+1) = B_i(t) + S_i(t) - R_i(t), \forall i, \forall t. \quad (5)$$

In terms of recharging energy, constraint $0 \leq S_i(t) \leq S_i^{max}$ and (4) can be combined into the following constraint,
$$0 \leq S_i(t) \leq \min\{S_i^{max}, B_i^{max} - B_i(t)\}. \quad (6)$$

Note that the recharging operation (occurring when $G_i(t) > D_i(t)$) and the discharging operation (occurring when $G_i(t) < D_i(t)$) of a battery will not happen simultaneously, which means
$$\begin{cases} S_i(t) > 0 \Rightarrow R_i(t) = 0, \forall i, t; \\ R_i(t) > 0 \Rightarrow S_i(t) = 0, \forall i, t. \end{cases} \quad (7)$$

Similarly, the energy flow between two users in one time slot can only be unidirectional (e.g., either from user i to user j or from user j to user i), which leads to
$$\begin{cases} B_{i,j}(t) > 0 \Rightarrow B_{j,i}(t) = 0, \forall i, t; \\ B_{j,i}(t) > 0 \Rightarrow B_{i,j}(t) = 0, \forall i, t. \end{cases} \quad (8)$$

F. Problem Formulation

Given the system model described in Section II, we focus on the problem of minimizing the total payment of a microgrid as a whole over time. The motivation of this problem is to promote the usage of renewable energy by exchanging energy within a microgrid, which can reduce the dependence on the macrogrid and increase the service reliability of the mircogrid. The total payment is made up by two parts: the money paid for purchasing electricity and the money paid for renting transmission lines, respectively. Here, we ignore the operation cost and power loss for recharging batteries and transmitting electricity in the microgrid, which can be easily extended based on our models. We mathematically formulate the problem of minimizing the time-averaged total payment of a microgrid as follows,

$$\begin{aligned} \min \quad & \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i=1}^{N} (p_i(t) E_i(t) + \sum_{j \neq i} q_i(t) B_{i,j}(t)) \\ \text{s.t.} \quad & (1), (2), (3), (6), (7), (8), \\ & \text{Battery queues } B_i(t) \text{ keep stable}, \forall i, \forall t. \end{aligned} \quad (9)$$

Problem (9) is an online stochastic optimizing problem. Without knowing future information, the central controller needs to control energy flows, by deciding the recharged energy $S_i(t)$ and discharged energy $R_i(t)$ of each user, the energy exchanged between any pair of users $B_{i,j}(t)$ and the electricity purchased from the macrogrid $E_i(t)$, according to the information collected in the current time slot such as $B_i(t), p_i(t), q_i(t), G_i(t)$ and $D_i(t)$.

The definition of queue stability is given in Definition 1.

Definition 1 (Queue Stability)**.** *A queue Θ is stable if and only if*

$$\lim_{T\to\infty} \sup \frac{1}{T}\sum_{t=0}^{T-1} \mathbb{E}(\Theta(\tau)) < \infty,$$

where $Q(t)$ is the backlog of the queue in time slot t.

III. Online Control Algorithm

In this section, we propose an online algorithm for the central controller of a microgrid to make controls on energy flows, aiming to minimize the total payment to the macrogrid. First, we reformulate the problem as a constrained linear programming solved in each time slot, assisted by the Lyapunov optimization technique. Then, we design an online algorithm based on several useful principles which give insights on the optimal solution of the reformulated problem. Finally, we prove that our proposed algorithm can achieve the approximate optimum with all constraints satisfied.

A. Problem reformulation

1) Virtual queues: We first transform actual battery queues into *virtual queues* for applying the Lyapunov optimization in the following. For each user, the virtual queue, denoted by Θ_i, is defined as follows,

$$\Theta_i(t) = B_i(t) - R_i^{max} - V p_i^{max}, \forall i, t, \qquad (10)$$

where V is a system constant parameter, with the value range $0 \le V \le V^{max} = \min_i\{\frac{B_i^{max} - S_i^{max} - R_i^{max}}{p_i^{max}}\}$. This parameter will be discussed in more detail in the following.

According to (10), we can easily find that battery queues are stable if and only if virtual queues are stable. Thus, the last constraint in Problem (9) can be equivalently replaced by the stability of virtual queues. According to (5), $\Theta_i(t)$ can be updated as

$$\Theta_i(t+1) = \Theta_i(t) + S_i(t) - R_i(t), \forall i, \forall t. \qquad (11)$$

Unlike actual battery queues existing in reality, only the backlog of virtual queues (the value of $\Theta_i(t)$) is maintained for operating our online algorithm. Moreover, the value of $\Theta_i(t)$ is not limited to be nonnegative.

2) Lyapunov optimization: In this subsection, we apply the theory of Lyapunov optimization to our problem, which provides a solution frame for stochastic programming problems. We first define the Lyapunov function based on the virtual queue matrix $\mathbf{\Theta}(t) = (\Theta_i(t))$ as follows,

$$L(\mathbf{\Theta}(t)) = \frac{1}{2}\sum_{i=1}^{N}(\Theta_i(t))^2. \qquad (12)$$

Then, the conditional Lyapunov drift $\Delta(\mathbf{\Theta}(t))$ can be defined to measure the expected shift of the Lyapunov function in one time slot as follows,

$$\Delta(\mathbf{\Theta}(t)) = \mathbb{E}\{L(\mathbf{\Theta}(t+1)) - L(\mathbf{\Theta}(t))|\mathbf{\Theta}(t)\}. \qquad (13)$$

The use of the expectation function is because of random variables $\Theta_i(t)$ in the online stochastic programming.

Based on the updating function of $\Theta_i(t)$ in (11), we can derive the upper bound of the conditional Lyapunov drift as shown in the following inequality,

$$\Delta(\mathbf{\Theta}(t)) \le C - \mathbb{E}\{\sum_{i=1}^{N}\Theta_i(t)(R_i(t) - S_i(t))|\mathbf{\Theta}(t)\}, \qquad (14)$$

where $C = \frac{1}{2}\sum_{i=1}^{N}[(S_i^{max})^2 + (R_i^{max})^2]$ is a constant. For completeness, the derivation process is given in the following.

Proof: According to the definition of the Lyapunov function and conditional Lyapunov drift, we have

$$\begin{aligned}\Delta(\mathbf{\Theta}(t)) &= \mathbb{E}\{L(\mathbf{\Theta}(t+1)) - L(\mathbf{\Theta}(t))|\mathbf{\Theta}(t)\}\\ &= \frac{1}{2}\mathbb{E}\{\sum_{i=1}^{N}[(\Theta_i(t+1))^2 - (\Theta_i(t))^2]|\mathbf{\Theta}(t)\}.\end{aligned}$$

By squaring both sides of (11), we can obtain

$$\begin{aligned}(\Theta_i(t+1))^2 &= (\Theta_i(t) + S_i(t) - R_i(t))^2\\ &= (\Theta_i(t))^2 - 2\Theta_i(t)(R_i(t) - S_i(t))\\ &\quad +(R_i(t) - S_i(t))^2\\ &\le (\Theta_i(t))^2 - 2\Theta_i(t)(R_i(t) - S_i(t))\\ &\quad +(S_i^{max})^2 + (R_i^{max})^2.\end{aligned}$$

By replacing $\Theta_i(t)$ with the right side of the inequality, we can derive the upper bound of $\Delta(\mathbf{\Theta}(t))$ as follows,

$$\begin{aligned}\Delta(\mathbf{\Theta}(t)) &\le \frac{1}{2}\mathbb{E}\{\sum_{i=1}^{N}[(S_i^{max})^2 + (R_i^{max})^2\\ &\quad -2\Theta_i(t)(R_i(t) - S_i(t))]|\mathbf{\Theta}(t)\}\\ &= \frac{1}{2}\sum_{i=1}^{N}[(S_i^{max})^2 + (R_i^{max})^2]\\ &\quad -\mathbb{E}\{\sum_{i=1}^{N}\Theta_i(t)(R_i(t) - S_i(t))|\mathbf{\Theta}(t)\}.\end{aligned}$$

We define the term $\frac{1}{2}\sum_{i=1}^{N}[(S_i^{max})^2 + (R_i^{max})^2] \triangleq C$, which is a constant given S_i^{max} and R_i^{max}. Therefore, we obtain the upper bound of $\Delta(\mathbf{\Theta}(t))$. ■

To minimize the conditional Lyapunov drift (for keeping queues stable) and minimize the total payment of a microgrid at the same time, we combine the two functions by applying the *drift-plus-penalty* method [7].

Definition 2. *The drift-plus-penalty function $\Delta_V(\mathbf{\Theta}(t))$ is defined as*

$$\begin{aligned}\Delta_V(\mathbf{\Theta}(t)) = \Delta(\mathbf{\Theta}(t)) &+ V\mathbb{E}\{\sum_{i=1}^{N}p_i(t)E_i(t) \qquad (15)\\ &+ \sum_{i=1}^{N}\sum_{j\neq i}q_i(t)B_{i,j}(t)|\mathbf{\Theta}(t)\},\end{aligned}$$

where $0 \le V \le V^{max}$ is the trade-off parameter balancing the queue stability performance and the total payment mini-

Algorithm 1 Online Control Algorithm for Total Payment Minimization

Input: Initializing $V \in [0, \min_i \{\frac{B_i^{max} - S_i^{max} - R_i^{max}}{p_i^{max}}\}]$

for each time slot $t = 0, 1, \cdots, T, \cdots$ **do**

Collecting information from each user including $B_i(t), G_i(t), D_i(t), p_i(t), q_i(t) \forall i$.

Solving the linear programming in (17) to obtain $S_i(t), R_i(t), E_i(t), B_{i,j}(t), \forall i$. Specially, the solution satisfies that $S_i(t) = 0$ when $B_i(t) > B_i^{max} - S_i^{max}$; $R_i(t) = 0$ when $B_i(t) < R_i^{max}$.

Updating virtual queues according to $\Theta_i(t+1) = \Theta_i(t) + S_i(t) - R_i(t), \forall i$.

end for

mization.

Remark: A larger value of V means minimizing the total payment is more important than keeping the queues stable. The central controller can tune the value of V, by comparing the significance of the two objectives.

3) Problem reformulation: In each time slot, given the values of $\Theta_i(t), p_i(t), G_i(t)$ and $D_i(t)$, the central controller can obtain the optimal decisions $S_i(t), R_i(t), B_{i,j}(t)$ and $E_i(t)$ by minimizing the drift-plus-penalty function in (15) subjecting to constraints (1),(2), (3), (6), (7) and (8). However, the computing complexity of this optimization problem is extremely high. To overcome the difficulty, we derive the upper bound of the drift-plus-penalty function by utilizing the upper bound of $\Delta(\mathbf{\Theta}(t))$ as follows,

$$\begin{aligned} \Delta_V(\mathbf{\Theta}(t)) \le C &+ V \sum_{i=1}^{N} \mathbb{E}\{p_i(t) W_i^-(t) | \mathbf{\Theta}(t)\} \\ &+ \sum_{i=1}^{N} \mathbb{E}\{\Theta_i(t) S_i(t) | \mathbf{\Theta}(t)\} \\ &- \sum_{i=1}^{N} \mathbb{E}\{[\Theta_i(t) + V p_i(t)] R_i(t) | \mathbf{\Theta}(t)\} \\ &+ V \sum_{i=1}^{N} \mathbb{E}\{\sum_{j \neq i} [q_j(t) - p_i(t)] B_{j,i}(t) | \mathbf{\Theta}(t)\} \quad . \end{aligned} \tag{16}$$

The detailed derivation is given as follows.

Proof: Substituting (14) into (15), we have

$$\begin{aligned} \Delta_V(\mathbf{\Theta}(t)) \le C + \mathbb{E}\{&- \sum_{i=1}^{N} \Theta_i(t)(R_i(t) - S_i(t)) \\ + V \sum_{i=1}^{N} p_i(t) E_i(t) &+ V \sum_{i=1}^{N} \sum_{j \neq i} q_i(t) B_{i,j}(t) | \mathbf{\Theta}(t)\} \end{aligned}$$

According to (3), we have $E_i(t) = W_i^-(t) - \sum_{j \neq i} B_{j,i}(t) - R_i(t)$. By replacing $E_i(t)$ in the right side of the above inequality, we can obtain

$$\begin{aligned} \Delta_V(\mathbf{\Theta}(t)) &\le C + \mathbb{E}\{\sum_{i=1}^{N} \Theta_i(t)(S_i(t) - R_i(t)) \\ &+ V \sum_{i=1}^{N} p_i(t) [W_i^-(t) - \sum_{j \neq i} B_{j,i}(t) - R_i(t)] \\ &+ V \sum_{i=1}^{N} \sum_{j \neq i} q_i(t) B_{i,j}(t) | \mathbf{\Theta}(t)\} \\ &= C + V \sum_{i=1}^{N} \mathbb{E}\{p_i(t) W_i^-(t) | \mathbf{\Theta}(t)\} \\ &+ \sum_{i=1}^{N} \mathbb{E}\{\Theta_i(t) S_i(t) | \mathbf{\Theta}(t)\} \\ &- \sum_{i=1}^{N} \mathbb{E}\{[\Theta_i(t) + V p_i(t)] R_i(t) | \mathbf{\Theta}(t)\} \\ &+ V \sum_{i=1}^{N} \mathbb{E}\{\sum_{j \neq i} [q_j(t) - p_i(t)] B_{j,i}(t) | \mathbf{\Theta}(t)\}. \end{aligned}$$

■

According to Lyapunov optimization, the central controller should make decisions in each time slot by minimizing the upper bound of the drift-plus-penalty function (the right side of (16)). Given the values of $\Theta_i(t), p_i(t)$ and $W_i^-(t)$, the problem can be reformulated through simplifying the right side of (16) as

$$\begin{aligned} \min \quad & \sum_{i=1}^{N} \Theta_i(t) S_i(t) - \sum_{i=1}^{N} [\Theta_i(t) + V p_i(t)] R_i(t) \\ & + V \sum_{i=1}^{N} \sum_{j \neq i} [q_j(t) - p_i(t)] B_{j,i}(t) \\ \text{s.t.} \quad & (1), (2), (3), (6), (7), (8), \end{aligned} \tag{17}$$

which is a constrained linear programming. By solving this problem, the central controller can obtain the optimal control decisions $S_i^*(t), R_i^*(t)$ and $B_{i,j}^*(t)$. Then, $E_i^*(t)$ can be calculated as $E_i^*(t) = \max(W_i^-(t) - R_i^*(t) - \sum_{j \neq i} B_{j,i}^*(t), 0)$.

B. Online algorithm design

As we have converted the problem of minimizing the time-averaged total payment into Problem (17) solved in each time slot, we propose an online control algorithm for the central controller in the microgrid, controlling energy flows just based on real-time information such as battery levels, energy generation and demand without future knowledge. In this section, we first give several useful principles solving the linear programming, and then propose an approximately optimal online control algorithm.

1) Principles of optimal solution: The operations of batteries (e.g., recharging and discharging) should follow Lemma 1.

Lemma 1. *When system parameter V is selected within the* *bound as follows,*

$$0 \le V \le \min_i \{ \frac{B_i^{max} - S_i^{max} - R_i^{max}}{p_i^{max}} \}, \quad (18)$$

there exist:

- *if* $B_i(t) > B_i^{max} - S_i^{max}$, *then* $S_i^*(t) = 0$*;*
- *if* $B_i(t) < R_i^{max}$, *then* $R_i^*(t) = 0$*.*

Proof: First, we focus on minimizing the first item of (17). It can be easily find that for each user i , $S_i^*(t)$ should be zero when $\Theta_i(t) > 0$. Substituting $\Theta_i(t)$ with (10), we have $S_i^*(t) = 0$ when

$$\begin{aligned} B_i(t) &> R_i^{max} + V p_i^{max} \\ &\ge R_i^{max} + \min_i \{ \frac{B_i^{max} - S_i^{max} - R_i^{max}}{p_i^{max}} \} \cdot p_i^{max} \\ &\ge R_i^{max} + \frac{B_i^{max} - S_i^{max} - R_i^{max}}{p_i^{max}} \cdot p_i^{max} \\ &= B_i^{max} - S_i^{max}. \end{aligned}$$

Note that the second inequality is obtained by using the upper bound of V as illustrated in Lemmea 1.

Second, we focus on minimizing the second item of (17). We can find that $R_i^*(t)$ should be zero when $\Theta_i(t) + V p_i(t) < 0$. Combining with the range of $p_i(t) \in [0, p_i^{max}]$, we can conclude that $R_i^*(t) = 0$ when $\Theta_i(t) < -V p_i^{max}$. Then we substitute $\Theta_i(t)$ with (10), and easily get that $R_i^*(t) = 0$ when $B_i(t) < R_i^{max}$. ■

Fortunately, Lemma 1 inspires the design of our online algorithm to achieve the optimal solutions of minimizing the upper bound of the drift-plus-penalty function. The results in this lemma is intuitive to understand. When a user has extra renewable energy beyond its own demand, it could transfer the energy to others if its battery level is over a threshold, i.e., $B_i^{max} - S_i^{max}$. On the other hand, when the renewable energy is not enough, a user tends to obtain energy from others or the macrogrid if the energy left in its battery is insufficient, like less than R_i^{max}.

2) Algorithm design for online control: The online control policy derived from our proposed algorithm is the optimal results of Problem (17). As Problem (17) is a linear programming with linear constraints, the central controller can solve the problem in polynomial time by using a linear optimization toolbox. With the insights from Lemma 1, we present our online control algorithm in Algorithm 1 in detail.

C. Optimization analysis

Finally, we prove that the constraints on battery queues in Problem (9) are satisfied over time, and then analyze the optimality of our proposed online algorithm in minimizing the time-averaged total payment of a microgrid.

Theorem 1. *Under our online algorithm, the bounds of battery levels,* $0 \le B_i(t) \le B_i^{max}, \forall i, \forall t$*, can be met for all users over time.*

Theorem 2. *The time-averaged total payment of a microgrid achieved by our online algorithm, denoted by* Ψ*, is approximately optimal, as shown in the following inequation,*

$$\Psi^* \le \Psi \le \Psi^* + C/V, \quad (19)$$

where Ψ^* *is the minimal payment of Problem* (9).

Theorem 2 demonstrates that the gap between the time-averaged result achieved by our online algorithm and the optimal result is less than a constant, which is associated with parameter V. A larger value of V implies our algorithm achieves better performance in minimizing the total payment. The proofs of these two theorems are omitted due to the limit of space.

IV. Performance Evaluation

In this section, we first describe the evaluation methodology and simulation setup, and then we demonstrate the performance of our proposed online control algorithm.

A. Methodology and setup

We compare the performance of our proposed algorithm with two heuristic algorithms, which serve as baselines. The two heuristic algorithms are designed as follows,

- **Heuristic Algorithm with conservative users (HA-CU)**: Here, we consider all users are conservative, which means user i will choose to charge its own battery full firstly if it has unused energy. In other words, $S_i(t) = 0$ only when $B_i(t) = B_i^{max}$. On the other hand, if user j lacks energy, it will choose to use up its battery firstly and then purchase energy from other users. In other words, $R_j(t) = 0$ only when $B_j(t) = 0$. The amount of exchanged energy $B_{i,j}(t)$ can be computed by minimizing $\sum_{i=1}^{N} \sum_{j \neq i} (q_i(t) - p_j(t)) B_{i,j}(t)$.
- **Heuristic Algorithm with obliging users (HA-OU)**: In this algorithm, we consider obliging users, who prefer transferring energy to others to storing in the battery. If user i is short of energy, it first discharges its own battery and then uses the excessive energy of other users as much as possible. The amount of exchanged energy $B_{i,j}(t)$ can be computed by minimizing $\sum_{i=1}^{N} \sum_{j \neq i} (q_i(t) - p_j(t)) B_{i,j}(t)$. Finally, user j will store energy unless there still is energy left after all users are satisfied.

The default values of parameters in our simulations are set as follows. We simulate a small-sized microgrid with 100 users. The time slot duration is 15 minutes, and all simulations last for 1000 time slots (more than ten days). The renewable energy generated in a time slot by each user is randomly distributed in $[10, 20]$kWh, while the demand is uniformly distributed in $[15, 30]$kWh. For simplicity, we set the maximal capacity of all batteries is 70kWh, and the maximal energy recharged or discharged in a time slot is 20kWh. In addition, the initial battery level of each user is zero. According to the real life, the dynamic price $p_i(t), \forall i$ of a unit of electricity from the macrogrid follows a uniform distribution over $\$[1, 3]$, while the price of renting electric transmission lines by user i, $q_i(t), \forall t$, is uniformly distributed in $\$[0.3, 0.6]$. The value of V is 10. We use the time-averaged total payment as our metric to measure the performance of the three algorithms.

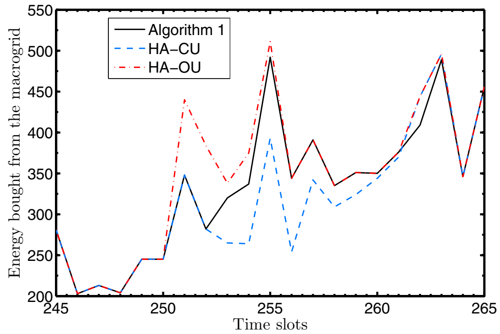

Fig. 2. The sum of energy purchased from the macrogrid in $t \in [245, 265]$.

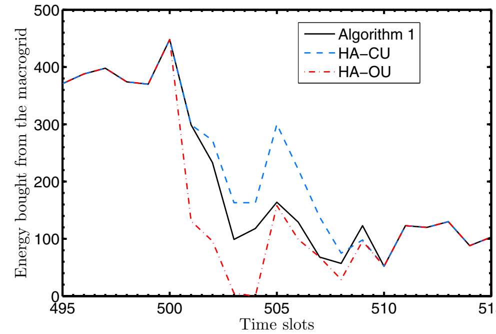

Fig. 3. The sum of energy purchased from the macrogrid in $t \in [495, 515]$.

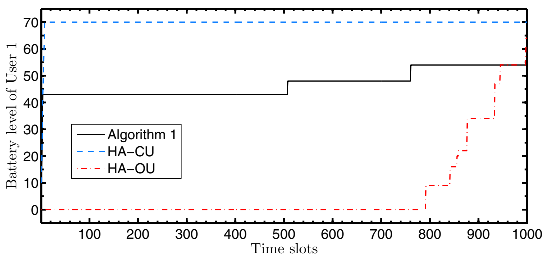

Fig. 4. The battery level of User 1 in each time slot.

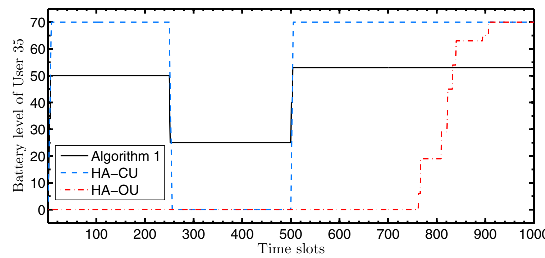

Fig. 5. The battery level of User 35 in each time slot.

B. Simulation results

1) Case study: We first simulate a special case, studying the responses of the three algorithms to the varying energy generation and demand. In this case, there are two categories of users: each user in one category is associated with generation $G_i(t) \sim U[20, 30], \forall i$ and demand $D_i(t) \sim U[10, 20], \forall i$, while each user in the other category is associated with generation $G_i(t) \sim U[10, 20], \forall i$ and demand $D_i(t) \sim U[20, 30], \forall i$. There are four stages over time with different percentages of the two categories of users, 250 time slots for each stage. The percentages of users in the first category of the four stages are 40%, 30%, 45% and 50%, respectively. We can find that the averaged amount of energy generated by all users first decreases from stage 1 to stage 2, and then increases significantly from stage 2 to stage 3, and finally rises gently from stage 3 to stage 4. The average demand of all users has an opposite variation. Moreover, the average generation is lower than demand in the first three stages and achieves balance in the last stage.

Fig. 2 and Fig. 3 plot the sum of energy purchased from the macrogrid according to the three algorithms in time slots $t \in [245, 265]$ and $t \in [495, 515]$, which cover the conversion from stage 1 to stage 2 and from stage 2 to stage 3, respectively. We can find that our proposed algorithm differs from the two heuristic algorithms when energy generation or demand vary significantly. When the energy shortage increases suddenly (e.g., $t = 250$), HA-OU suffers a sharp increase in energy purchased from the macrogrid as shown in Fig. 2. This is because excessive renewable energy is always transferred to others in previous time slots, leading to little energy stored in batteries. When the energy shortage decreases (e.g., $t = 500$), the purchased energy by HA-CU reaches a peak subsequently as presented in Fig. 3. This is because users with non-full batteries charge their batteries first when they have unused energy, and thus other users have to purchase energy from the macrogrid.

We also plot the variations of two users' battery levels over all time slots of the case study in Fig. 4 and Fig. 5, respectively. Note that User 1 is associated with $G_1(t) \sim U[20, 30]$ and $D_1(t) \sim U[10, 20]$ in the four stages. User 35 is as same as User 1 except being associated with $G_{35}(t) \sim U[10, 20]$ and $D_{35} \sim U[20, 30]$ in the second stage. We can find that the battery levels achieved by our proposed algorithm demonstrate Lemma 1, as $B_1(t) \leq B_1^{max} - S_1^{max} = 50$ in the four stages and $B_{35}(t) \geq R_i^{max} = 20$ in the second stage. In HA-CU, the battery level of any user increases to the maximum when its average generation is larger than its average demand and drops to zero on the contrary. In HA-OU, the battery level of any user gradually rises only when the average generation of all users is larger the the total average demand. Generally, our algorithm has more gradual variations in battery levels than HA-CU and HA-OU.

2) Performance comparison: Next, we compare the performance of the three algorithms in the time-averaged total payment. We perform twenty runs with different random seeds for the default setting. The results are reported in Table I. We can find that the average performance of our algorithm is 6.2%

TABLE I
PERFORMANCE ON TIME-AVERAGED TOTAL PAYMENT

	mean	95% confidence interval
Algorithm1	86.345	[83.996, 88.694]
HA-CU	91.699	[89.507, 93.892]
HA-OU	103.179	[100.374, 105.984]

and 19.5% higher than HA-CU and HA-OU, respectively. Actually, our algorithm performs much better than the other two algorithms, as we do not take the energy stored in batteries at the end of simulation into consideration when computing the performance. According to Fig. 4 and Fig. 5, the energy left in batteries achieved by our algorithm is much more than HA-CU and HA-OU.

V. RELATED WORK

Recently, the concept of smart grid becomes increasingly popular. In this section, we review some related works briefly, and compare the differences between them and our work.

Smart grid: As the smart grid has a promising future to replace the traditional power grid, there are increasing research efforts recently focusing on the smart grid. Comprehensive surveys [8][9] on the smart grid can be found. These surveys not only summarize the existing works, but also indicate the valuable directions of the future research. Accordingly, the smart grid is divided into three major systems, including the infrastructure system, the protection system and the management system. Moreover, the conceptual smart grid gradually comes true due to the fast development of distributed generation [1], energy storage [2], smart meters [10] and communication networks [11]. In particular, microgrid [12] and vehicle-to-grid (V2G) [13] are seen as two of the most significant paradigms of the future smart grid.

Renewable energy efficiency: Considering the variation in renewable energy production, a series of works have focused on minimizing energy loss of a whole system. In [3], the authors aim to minimize the operation cost and guarantee the quality of service in electricity (QoSE) in a microgrid. In their assumptions, residential usage is divided into basic usage and quality usage, where basic usage can be covered by the minimum capacity of the microgrid and an outage of quality usage can be tolerated partly. Both [4] and [5] consider renewable energy exchanging between different microgrids. [4] analyzes the tradeoff between two techniques: the use of storage and cooperated distributed generators. It considers transferring energy between two microgrids or from the macrogrid to a microgrid with a cost. In [5], the total power losses of the entire smart grid, including the costs of charging and discharging batteries and the power losses during transfer, are optimally reduced. This work assumes that the power loss between two microgrids is related to their distance.

VI. CONCLUSION

In this paper, we have focused on the vital yet challenging problem of maximizing the long-term renewable energy usage via exchanging excessive renewable energy among different users in a microgrid. We model the problem as a constrained optimization problem, minimizing the time-averaged total payment to the macrogrid. An online control algorithm is proposed for the central controller, which achieves the approximate optimal performance through managing batteries of users, energy exchange within the microgrid and energy purchased from the macrogrid. Comprehensive simulations have been performed, which validate the superiority of our online control algorithm. In terms of the time-averaged total payment to the macrogrid, our algorithm saves 6.2% and 19.5% compared with HA-CU and HA-OU, respectively.

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