Explore Hidden Information for Indoor Floor Plan Construction

Bing Zhou
Department of Electrical and Computer Engineering
Stony Brook University
Stony Brook, NY 11790
Email: bing.zhou@stonybrook.edu

Fan Ye
Department of Electrical and Computer Engineering
Stony Brook University
Stony Brook, NY 11790
Email: fan.ye@stonybrook.edu

***Abstract*—The lack of digital floor plans in most buildings has become a huge obstacle to pervasive indoor location based services (LBS). Recently there has been quite some research that leverages various sensing data such as inertial, WiFi and images from ubiquitous mobile devices (e.g., smartphones) to construct floor plans at large scale and low costs. Although great efforts are made to improve the accuracy and robustness against sensing data errors and noises, the quality of reconstructed maps is still limited. In this paper, we explore the hidden geometric structure information of indoor environments, such as collinearity of doors along hallways, right-angle corners, and polygon/circular shapes of rooms to optimize floor plans. Such prior knowledge about building structures provide new spatial relationships among floor plan elements. Thus we can further improve the quality of reconstructed maps. Real experiments in two large buildings show that 90-percentile landmark location errors are reduced by more than 50% to within 1m, and most orientation errors are corrected. The overall shape of the map has become much closer to the ground truth as well.**

I. Introduction

Online digital maps (e.g., Google Maps) has provided great convenience for location based services (LBS) outdoors such as finding nearby point-of-interests (POIs) and navigation. However, for indoor environments where people spend over 80% of time [1], such maps are extremely scarce and most buildings simply do not have indoor digital maps. This has become a huge obstacle to pervasive LBS indoors.

Accurate, scalable construction of indoor floor plans at low costs has become an urgent need. Autonomous robots equipped with various high precision special sensors (e.g., laser rangers [2], depth cameras [3], sonars [4]) can create high quality maps. However, due to the high manufacturing costs, operational and logistic obstacles deploying robots in large quantities, they do not satisfy scalability and costs requirements. Recently there has been quite some work [5]–[7] that crowdsources various sensing data from commodity mobile devices to achieve scalable and low cost map construction. A representative one, Jigsaw [6], combines inertial and image data to produce user trajectories and geometry attributes of indoor landmarks (e.g., doors, posters on walls). It then fuses all such information as spatial constraints among elements (hallways, rooms) to produce complete floor plans.

In this work, we explore a previously ignored dimension called structure cues, the common structural relationships among indoor elements, to further improve the quality of constructed maps. Unlike existing work that derives spatial constraints from sensor measurements only, we leverage prior structural knowledge about general rules in floor plan layout. In particular, we examine three basic types of structure cues: collinear wall segments, right-angle corners, and polygon/circular room shapes. They appear regularly in many kinds of buildings such as offices and provide additional constraints delineating the locations, orientations, shapes and sizes of indoor elements. Thus we can further improve the quality of constructed maps.

We make the following contributions in this work:

- We explore three common structure cues including collinear wall segments, right-angle corners and polygon/circular rooms. We design three corresponding algorithms for detecting these structures: one density based clustering algorithm for collinear wall segments, one threshold based method for right-angle corner detection, and one to detect polygon/circular rooms using minimum out-bounding shapes.
- We design an algorithm using majority voting on landmarks within the same clusters to find the optimal fitting model for hallways. Compared to traditional regression methods, it can merge large numbers of landmark samples in a probabilistic estimation. We also design algorithms for geometric parameter estimation: One estimates the starting and ending points of hallway segments based on user traces and landmark locations, another to find out-bounding polygon/circular shapes for room representation.
- We conduct experiments in two large real buildings of size $80{\times}50\text{m}^2$ and $90{\times}50\text{m}^2$. We find that the 90-percentile landmark location estimation errors are cut by more than 50% to within 1m, and orientation errors are almost all corrected. The overall shapes of maps are also become much closer to respective ground truth.

The rest of this paper is organized as follows: Section II presents background of particle filter based landmark mapping, which is prior work that produces maps. In Section III, we introduce an overview of the system design for map optimization. Section IV presents algorithms for structure detection

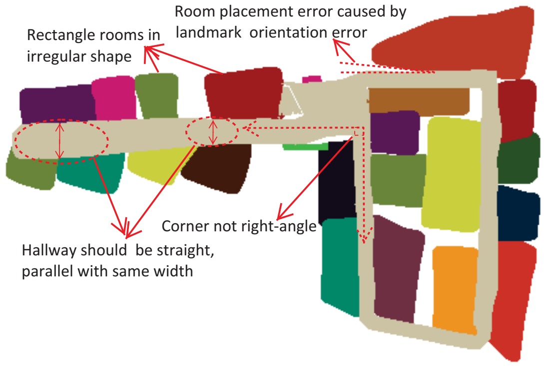


Fig. 1. Common errors in input inaccurate maps for hallways and rooms.

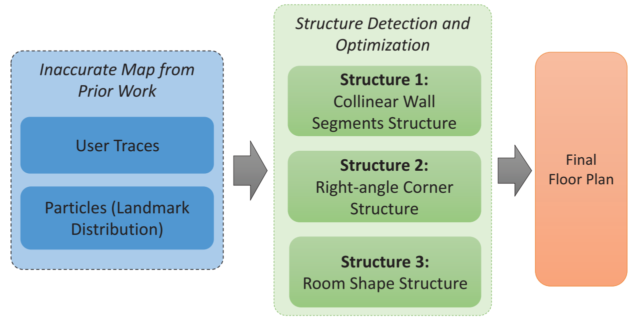


Fig. 2. Prior work builds map from user traces and particles, which is inaccurate. Our system takes inaccurate map as input and leverages three structure cues to optimize final map.

and map optimization. We evaluate the mapping improvement performance in Section V. The rest sections are related work and our conclusion.

II. Background

We introduce some necessary background of a probabilistic landmark mapping algorithm using particle filters [8], which can generate maps that potentially can be improved. Our paper leverages structure cues to improve such maps, which are constructed from the probability distribution of attributes (e.g., locations, orientations) of landmarks in the form of "particles" and user traces. Each particle is a sample with a probability for the complete internal states of a map, including values for attributes of all landmarks.

The algorithm progressively updates landmark attributes such as locations/orientations as new measurement data come in. A poster or door on a wall is a landmark described by its location, orientation, and lengths of two adjacent wall segments. Let $L = \{l_1, l_2, ..., l_N\}$ be the complete set of landmarks for one particle. Each landmark l_i is described as a five dimensional vector:

$$l_i = (x, y, \phi, w_L, w_R) \in R \times R \times [0, \pi) \times R \times R \quad (1)$$

where (x, y) are coordinates of its location, ϕ is its orientation, w_L and w_R are lengths of two adjacent wall segments next to the landmark.

Suppose we have M particles and $P = \{p_1, p_2, ...p_M\}$ is the particle set. Each particle p_j is represented as $\{l_1^j, l_2^j, ...l_N^j\}$, each l_i^j is a landmark estimator that is maintained and updated during the mapping. In Figure 3(a), each physical landmark has M samples, each visually represented as a line segment and a circle to denote its orientation and location.

Combining user traces from inertial data and landmark samples, the sizes and shapes of hallways and rooms can be inferred using occupancy grid mapping [6] to form complete floor plan. However, as landmarks are maintained and updated individually and probabilistically, samples vary greatly in accuracy. Thus errors in hallway and room sizes, shapes can all happen, e.g., non-straight hallways, turns with large angle errors, and rooms in irregular shapes, as shown in Figure 1. This paper leverages structure cues to correct such errors for accurate, complete maps.

III. System Overview

A floor plan consists of two types of basic elements: hallways and rooms. Hallways form the overall structure of map layout. They are inferred from user traces and landmark samples, and rooms are placed with corresponding landmarks (e.g., doors) for the final map. This paper focuses on how to leverage structure cues to improve and optimize hallway/room layout and shapes, given user traces and landmark estimations from the particle filter. Figure 2 shows that three structure detection and optimization techniques take inaccurate input maps to produce accurate final maps.

We leverage three structure cues for map optimization:

1) Collinear Wall Segments Structure. It includes three steps: cluster landmark samples into collinear groups belonging to the same walls based on their orientations and locations; merge collinear landmark samples to form hallways based on a majority voting mechanism; combine user traces and landmark samples to estimate the lengths of hallways.

2) Right-angle Corner Structure. It detects and estimates angles between intersecting hallways (in our case, the right-angle corners), and makes global adjustments for hallway orientation under Manhattan world model [9].

3)Room Shape Structure. It detects common shape patterns in most indoor environments, e.g., rectangle, polygon or circular shapes. The fitted shapes are computed to represent rooms after the detection.

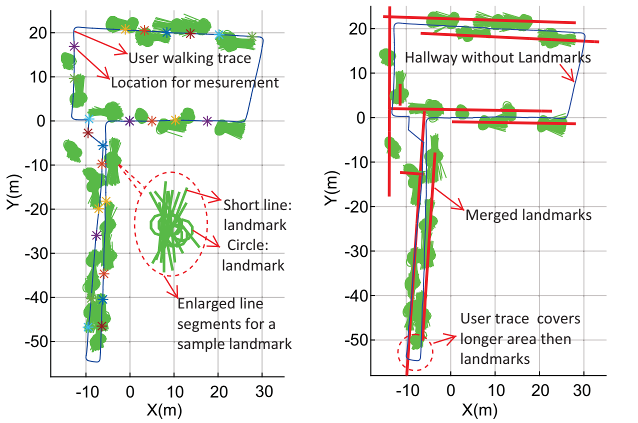
IV. Structure Detection and Optimization

A. *STRUCTURE 1: Collinear Wall Segments Structure*

Particle filter produces landmark samples as short line segments (wall segments), and each landmark l_i has hundreds of samples as shown in Figure 3(a). We need to cluster these wall segments into groups belonging to the same walls, then merge those in each group into single walls and estimate the location/orientation/lengths of these walls to form hallways.

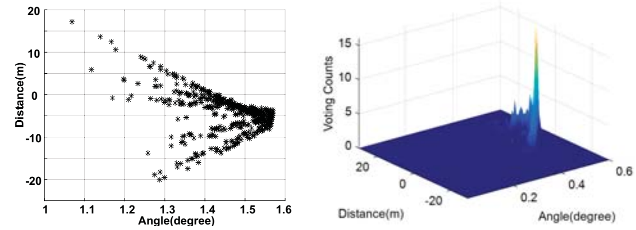
1) Landmark Clustering

We cluster landmarks into collinear groups using each landmark's sample with the highest probability. Based on measurement evidences, they are supposed to be the ones closest to the ground truth. Clustering using all samples of the landmarks is difficult, because each landmark's samples



(a) User trace and landmarks (b) Line segments merging

Fig. 3. a) long trajectory is user trace, stars are user locations for measurement, short line segments are landmark estimations. b) long line segments are merged wall segments.



(a) Each star corresponds to each line segment (b) Voting for line segment

Fig. 4. a) shows the polar representations of landmark with in the same cluster, b) shows the majority voting in 3-D perspective.

may vary greatly in orientation, thus a dominant orientation may not exist.

The clustering process has two steps: cluster landmarks based on similar orientations, then further on different walls, using the highest probability sample for each landmark. These landmarks belong to parallel walls of parallel hallways. The second step distinguishes landmark samples on different parallel walls. DBSCAN [10] is applied for both orientation and distance clustering. Unlike k-means [11] which needs to specify the exact number of clusters, it needs only a threshold and does not require the number of clusters as prior knowledge. Line segments of the highest probability for each landmark are clustered into different groups, each represented by a common orientation estimation θ_i, which is the mean of the cluster; within each group, we further cluster the line segments based on the origin point's perpendicular distance to the line, defined as:

$$d = x\cos\theta_i + y\sin\theta_i \tag{2}$$

where (x, y) is the location of landmarks, and we assign the same orientation value θ_i for all line segments in the same collinear group, hence eliminating orientation variations in samples. Clustering on θ determines which landmarks are parallel but they can belong to different parallel hallways. Clustering on d further distinguishes parallel wall segments, and finally divide landmarks into groups belonging to the same walls.

Existing work [12] finds a base line at first step and then group line segments along each base line, Rolf *et al.* [13] detects locally common directions and then cluster line segments by collinearity. However they can not be applied directly here. Because each single landmark has many samples with large variations in orientations. These methods perform clustering over all line segments, hence samples of different landmarks may be mixed together. Thus a dominate orientation does not necessarily exist, finding a base line or locally common direction becomes difficult.

2) Landmark Merging

We leverage majority voting among each group of collinear landmarks on the same wall to find the single fitting line as estimation of location/orientation of the wall. Within each group, we compute the orientation θ and distance ρ for all samples of each landmark, and distribute them on a 2-D grid plane shown as Figure 4(a). The area with densest points in Figure 4(a) corresponds to the peak in Figure 4(b) when majority voting is applied. Each point represents a support sample for a pair (θ, ρ). We use the orientation and distance represented by the peak in Figure 4(b) as the polar parameters for the collinear line with the most supporting samples. This enables us to find the common structures within all the particles states instead of using particles with highest probability only.

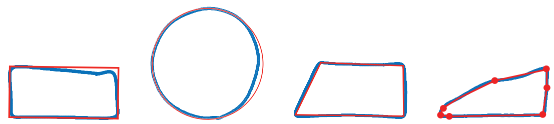
We do not use common regression methods such as least square to fit highest probability landmark samples only or the whole set of landmark samples. Such methods do not minimize errors unless errors cancel out each other among all landmark samples. As hundreds of particles are used in the landmark mapping algorithm, we can assume there must be landmark samples very close to ground truth. Structure cues give us a way to find out those “correct” landmark samples, and finally get the fitting line. Finding the fitting line supported by most particles is more reasonable because they have the most colinear line segments, thus more likely to correspond to ground truth straight lines.

3) Hallway Length Estimation

In the previous section, landmarks are clustered into walls. In Figure 3(b), the long line segments represent side walls of hallways. However they only give orientations but not exact length estimations. We estimate hallway lengths according to the following steps, by leveraging the following additional prior knowledge in a sequential order:

1) Landmarks. Merged hallway should cover the whole set of landmarks within it, hence a minimum hallway length can be estimated based on landmark locations and lengths. However it is far from enough because the wall segments rarely extend to both ends of a hallway, and in some cases we may not have any landmarks along a hallway at all (as shown in Figure 3(b)).

2) Traces. We leverage user walking traces to further adjust hallway length estimation. User traces indicate the accessible areas in hallways, hence hallway lengths are extended to cover the accessible areas that are inferred from traces. One typical example is the long hallway in Figure 3(b), user trace



(a) Rectangle (b) Circle (c) Polygon (d) Irregular

Fig. 5. Different kinds of room reshaping: minimum out-bounding rectangle/circle fitting (a,b), and turning points connection fitting (c,d).

covers longer area than landmarks, hence the hallway length is extended to cover the user trace.

3) Crossing and Parallelism. Adjacent hallways should be connected to each other. We use the wall connection algorithm in Jigsaw [6] to find intersecting points between hallways. Then we adjust the orientations of walls on both sides of the same hallway to their mean value to make them parallel.

B. STRUCTURE 2: Right-angle Corner Structure

The limited accuracy and long term drift of gyroscope make it hard to track the orientation accurately, especially during turns [14]. A few degrees' error in angle estimation can cause large location errors of a long hallway after the turn, thus the serious error in hallway layout and final map. The way a user holding the phone is also a main contributor to large errors. From our experiment experiences, users tend to swing their hands/arms during walking, causing significant errors in orientation estimation.

1) Right-angle Corner Detection

Manhattan [9] structured buildings are quite common in real world. In such buildings, corners are mostly right angled, such right-angle turns provide opportunities that can be used for calibration. We compute the angle between each two intersecting wall segments. A threshold method is used to detect right-angle corners: if the computed angle is within a small offset (5° in our case) to 90°, the intersecting corner is detected and adjusted as right-angle corner.

2) Hallway Orientation Calibration

Based on detected right-angle corners, the algorithm performs adjustments on orientations of the associated hallways gradually. The orientation of the longest hallway is assumed to be that of the global coordinate system.The algorithm rotates hallways around corner intersecting points to make them parallel to global horizontal or vertical orientations. For those none right-angle corners, the orientation remains the same.

C. STRUCTURE 3: Room Shape Structure

We use two techniques for room reshaping: out-bounding rectangle fitting and turning points connection. After that, each room is placed to respective hallway using the associated landmark (i.e., door) location to formulate the final map. We have three steps for room optimization: room shape detection, room reshaping, and room arrangement.

1) Room Shape Detection

The prior knowledge of rooms (e.g., rectangle, polygon, circular) can be leveraged to optimize reconstructed room shapes. Users walk around the perimeter of each room's interior, then trajectories are constructed from inertial data to infer rough room shapes. However, this simple method has limitations: user trajectories may be blocked by obstacles inside rooms, and users cannot walk close to inner walls exactly. Usually traces tend to be smaller than the actual size, hence causing false negative estimations. We detect room shapes as rectangles and circles, which are most common in indoor environments. The following steps describes the detailed steps:

1. Generate user trajectories from inertial data to represent raw room shapes $\{R_1, R_2, ...R_N\}$.

2. Minimum bounding box algorithm [15] is used to compute the minimum out-bounding rectangle for each room R_i. The ratio between raw room size and minimum rectangle size is denoted as r_rec. If $r\_rec > 0.9$, which means the shapes are similar, then the room is classified as rectangle.

3. We use an existing algorithm [16] to compute the smallest out-bounding circle for the remaining rooms and compute the size ratio r_cir between raw room and out-bounding circle. Then we use the same threshold to detect circular rooms.

4. For those unclassified rooms, we represent such rooms with polygons. r_rec is used to determine how many edges are needed to fit the room shape in the next step.

2) Room Reshaping

For rectangle/circular rooms, we use the same minimum out-bounding rectangle/circle computed in the detection step to fit the room shape. In this way, room corners and inaccessible areas can be fully reconstructed, such as Figure 5(a) and 5(b).

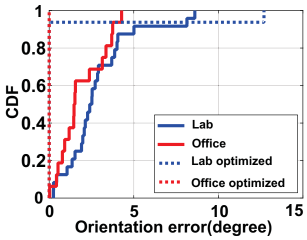
For those unclassified rooms, we fit the room shapes by connecting turning points on the trajectory. We take the derivative of user walking orientation, and find the peaks of the absolute values of derivatives and sort them in descending order. These peaks are defined as turning points. We use the round down number $\lfloor 4/r\_rec \rfloor$ to determine how many turning points are used for fitting. In this case, if $r\_rec > 0.8$, the room will be regarded as rectangle/quadrangle and four turning points will be used, such as Figure 5(a) and 5(c). If $r\_rec <= 0.8$, it means the shape could be irregular, which is far from a rectangle. In this case, more turning points are used to make better fitting, as shown in Figure 5(d).

3) Room Arrangement

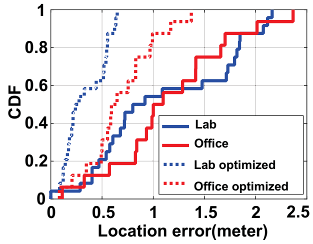
We place each reshaped room to the corresponding position along the hallway, according to the corresponding landmark location and orientation. Since users start walking a closed trajectory from the door location, hence room location can be determined by align door locations to the corresponding landmark and rotating room to achieve minimum overlap between room shape and hallway.

V. EXPERIMENTS AND RESULTS

To evaluate the performance of the design, We conduct experiments on the data set from two indoor environments: a 90×50m^2 office and an 80×50m^2 lab building, and compare the inaccurate input maps from prior work. There are 16 and 24 landmarks in office and lab, respectively. The number of landmarks depends on the complexity of the actual



(a) Landmark Orientation Error



(b) Landmark Location Error

Fig. 6. Landmark location and orientation error before and after optimization.

environment. In the particle filter algorithm, 500 particles are used. Thus we have a total of $16 \times 500 = 8000$ and $24 \times 500 = 12000$ line segments (landmark samples) to cluster and merge on two buildings. We evaluate the performance using the following metrics: landmark location and orientation error, hallway overall shape, and final map overall shape.

A. Landmark Location and Orientation

Landmark location and orientation are the fundamental measurement results for floor plan reconstruction. Wall connection and room placement rely on the location and orientation of landmarks, any deviation of landmarks will cause inaccurate hallway construction and room placement. Although particle filters can suppress certain noises and errors, its probabilistic nature means uncertainty will always exist. Besides, as landmarks are maintained and updated separately, the final landmarks will not follow internal structure relationships exactly, such as collinearity.

Figure 6 shows CDFs of landmark location and orientation errors for two buildings before and after optimization. In Figure 6(a), orientation errors in office are completely corrected. This is not surprising because office is Manhattan structured. The hallways are always perpendicular to each other which gives us perfect opportunities to correct angle errors. In the lab, most orientation errors are corrected except one outlier, which is a false correction of a single landmark in a group without other landmarks to compensate. Figure 6(b) shows the landmark location errors are reduced from around 2 meters at 90-percentile to less than 1 meter. The maximum location error in the lab is reduced from more than 2 meters to around 0.5 meters.

B. Hallway Shape Evaluation

Robust and accurate hallway construction is crucial to final map quality. We evaluate the hallway shape separately for the two buildings and compare with the inaccurate input maps. Figure 7 shows the ground truth, inaccurate input hallways, and optimized hallways for both buildings. To quantify the overall shape accuracy, precision, recall and F-score are defined below to measure the similarity to ground truth:

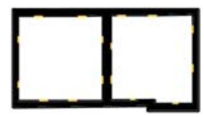
$$P = \frac{S_{re} \cap S_{gt}}{S_{re}}, R = \frac{S_{re} \cap S_{gt}}{S_{gt}}, F = \frac{2P \cdot R}{P + R}, \tag{3}$$

where S_{re} denotes the size of reconstructed map, S_{gt} that of its ground truth, and $S_{re} \cap S_{gt}$ that of the overlapping area.

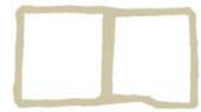
(a) Lab hallway ground truth

(b) Lab hallway of inaccurate input map

(c) Lab hallway with structure cue optimization



(d) Office hallway ground truth



(e) Office hallway of inaccurate input map

(f) Office hallway with structure cue optimization

Fig. 7. Hallway ground truth, inaccurate input hallways and structure cue optimization results.

A perfectly reconstructed map should have 100% precision, recall and F-score.

Table I shows the comparison results of input inaccurate maps and optimized maps for lab and office hallways. For the lab building, both precision and recall are increased by $\sim 20\%$, hence we get a higher F-Score. The main reason is that the long skewed hallway is calibrated after wall segments merging and right-angle turn calibration. For the office building, we get higher precision and F-score while there is a small decrease ($\sim 2\%$) in recall. This decrease is caused by false line segments clustering, hence the hallway in the bottom which has two segments are merged into one.

TABLE I
SHAPE EVALUATION OF HALLWAY

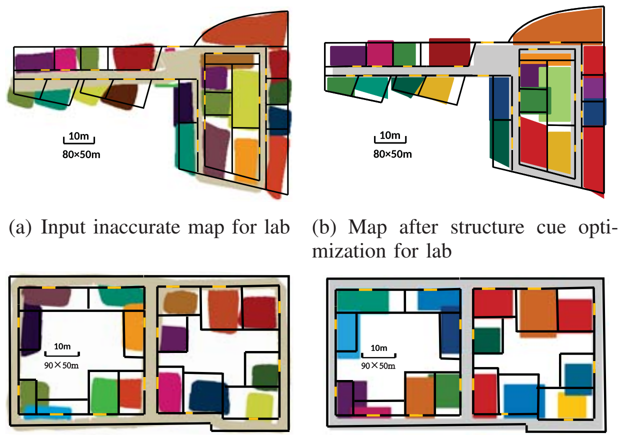
	Precision	Recall	F-Score
Lab Input	67.44%	54.80%	60.46%
Lab Optimized	84.86%	75.86%	79.93%
Office Input	78.28%	81.75%	79.97%
Office Optimized	85.53%	79.50%	82.40%

C. Overall Map Shape

We evaluate the final map shape after reshaped room are added to hallways. To make it easier to compare, we overlay the constructed map onto the ground truth to achieve the maximum overlap by rotation and translation, which is shown in Figure 8. We can see obvious improvement in maps after structure cue optimization: they match the ground truth much better. Precision, recall and F-score are again used for evaluating overall shapes and the results are shown in Table II. We get improvements in precision, recall and F-Score in both two buildings except a sightly recall decrease ($\sim 2.5\%$) in the office, which is caused by missed detection of small turn angle on the right bottom in Figure 8(c). However the final maps are improved overall, we have up to 4% increase in F-score on both buildings.

TABLE II
SHAPE EVALUATION OF FLOOR PLANS

	Precision	Recall	F-Score
Lab Input	87.73%	85.51%	86.61%
Lab Optimized	89.27%	92.23%	90.72%
Office Input	75.16%	95.96%	84.30%
Office Optimized	83.72%	93.42%	88.30%



(a) Input inaccurate map for lab (b) Map after structure cue optimization for lab

(c) Input inaccurate map for office (d) Map after structure cue optimization for office

Fig. 8. Final maps of two buildings with ground truth overlaid both for inaccurate input maps and maps after structure cue optimization.

In Figure 8(a), we can see the obvious skew for the long hallway and the associated rooms. The largest room on the top right and the one on the bottom right corner have obvious orientation errors caused by landmark orientation errors. Both kinds of errors are eliminated with structure cue optimization, as shown in Figure 8(b). In Figure 8(b), most rooms are detected as rectangles, which matches the ground truth very well; a few polygons and irregular shape room are also fitted closer to ground truth. In Figure 8(d), room shape optimization results are much better as all the rooms in office building are rectangles. The final map looks much closer to ground truth.

VI. Related Work

Indoor Floor Plans. Indoor floor plan construction is becoming an urgent and active research problem. The existing work has used mobile sensing techniques and crowd-sourcing. CrowdInside [5] uses inertial data to approximate shapes of accessible areas. Jigsaw [6] combines vision and mobile techniques to generate floor plans. Jiang *et al.* [7] leverage Wi-Fi signatures to detect room and hallway adjacency, and combine user trajectories to construct hallways. All the above work can approximate the rough shapes of actual floor plan. However, none of them has explored the structure cues for map optimization.

SLAM. (Simultaneous Localization And Mapping) is a famous problem in robotics, which refers to constructing a map of an unknown environment while tracking the robot's location on the map. A lot of work has used special hardware such as laser rangers [2], depth cameras [3], sonars [4], which limit the applicability and scale. Some recent work [17], [18] has used sensors in mobile devices but mostly focuses on localization, not map construction.

VII. Conclusion and Future Work

In this paper, we explore the structure cues in indoor environments and leverage such prior information to optimize floor plans. Results show that structure cues are effective in improving the shapes/sizes of hallways and rooms for much improved final maps.

The main limitation of this work is the generality of structure cues in complex indoor environments, such as big shopping malls, train stations. They may not have such rich and obvious structure cues to follow. We believe there exists more general prior information on their structures, and will explore different kinds of buildings to further identify new types of structures cues.

VIII. Acknowledgement

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